Correction of measured X-ray spectra by an analytical response matrix using Matlab.

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Abstract:
The X-ray spectrum produced by the X-ray tube and displayed by the spectrometer, the so-called instrumental spectrum, differs from the real, physical energy beam spectrum due to the physically inevitable instrumental artefacts. For the common X-ray energies used in the diagnostic radiology it is necessary to take into account the following artefacts: i) the energy dependence of the detection efficiency; ii) the escape effects; iii) the Compton effect; iv) the incomplete charge collection from the detector volume.
Various types of artefacts that are present in the measured (instrumental) spectra can be partially eliminated using the response matrix of the spectrometric system. The response matrix can be constructed for a specific detector from the known probabilities of processes leading to the artefacts in the measured spectra.

Method:
Let ***A*** be a vector describing the real, physical energy spectrum of the measured X-ray beam. Each element A_i represents the relative number of photons with the energy in the interval $<E_j; E_j+\Delta E)$. It holds $E_1 = 0$, and $E_{j+1} = E_j + \Delta E$. Let ***B*** be a vector describing the instrumental spectrum displayed by the measuring device, B_k being the relative number of photons detected in the energy interval $<E_k; E_k+\Delta E)$.
In the process of measurement, the initial (unknown) physical spectrum, ***A***, is transformed to the measured instrumental spectrum, ***B***, what can be expressed as:

$$\boldsymbol{B} = \boldsymbol{A}.\boldsymbol{R}, \tag{1}$$

where $R(E_j, E_k)$ is the so-called *response matrix* of the detector. The response matrix is a relatively complex triangular matrix which is detector specific and can be in principle calculated using the knowledge base and the formal apparatus of the atomic and nuclear physics (Johns and Cunningham, 1983; Redus et al., 2009; Miyajima, 2003). In the diagnostic radiology X-ray spectra, the suitable number of vector elements B_k is around 300. Providing the response matrix ***R*** is known, the original physical spectrum, ***A***, can be reconstructed from the measured instrumental spectrum, ***B***:

$$\boldsymbol{A} = \boldsymbol{B}.\boldsymbol{R}^{-1}, \tag{2}$$

In real situations, the response matrix can be enumerated only approximately, also the measured counts B_k are subject to the measurement uncertainty and the statistical uncertainty, which results in some computational challenges.

Results and discussion:
Beams of the N series (ISO 4037-1:1996) were realized using the X-ray unit Isovolt 320 Titan (GE Inspection Technologies, GmbH, Ahrensburg). Filtrations corresponding to the N (narrow) series spectra were settled according to the norm (ISO 4037-1:1996). Measurement of the spectra was performed using the Amptek CdTe XR-100T spectrometer. Response matrix ***R*** was constructed in Matlab and applied to the measured spectra (see Figures 1-3). Suitable parameters for characterization of the spectra (see Tables 1 and 2) were derived and compared with the parameters cited in the norm (ISO 4037-1:1996).

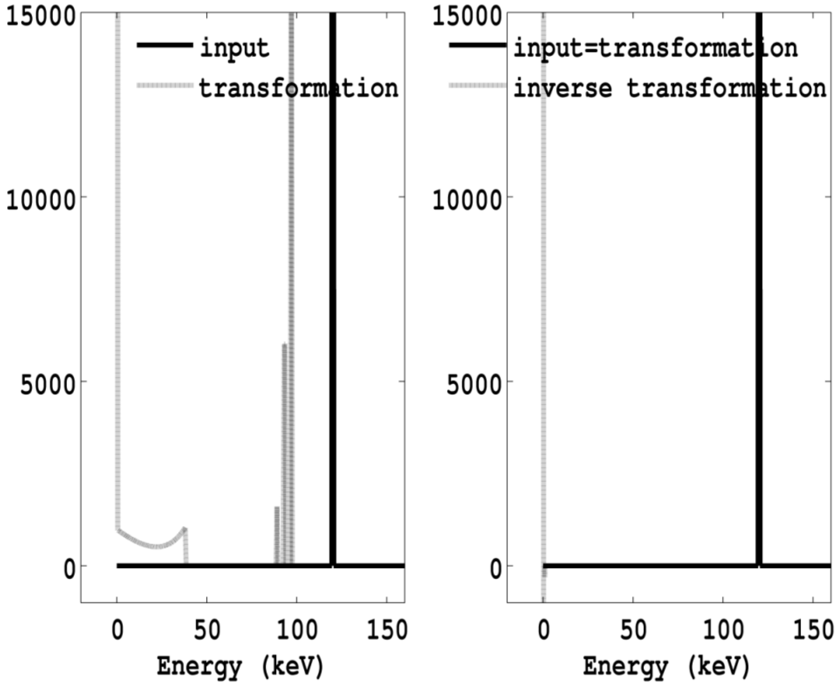

Figure 1. Left: The application of the response matrix ***R*** on a hypothetical 120 keV single line spectrum ***A*** with 10^6counts. In the transformed spectra ***B*** we can see signals due to the escaped photons, the Compton continuum, and also the virtual signal corresponding to the non-detected photons in the channel with energy E=0 keV. Right: The original single line spectrum ***A*** is recovered when the $\boldsymbol{R}^{-1}$ matrix is applied to the transformed spectrum ***B***.

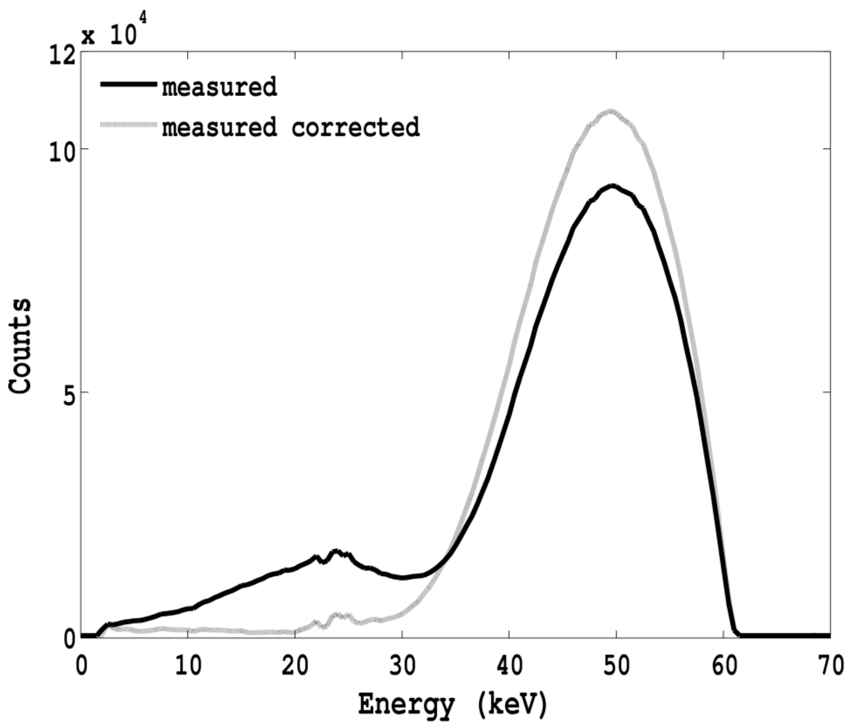

Figure 2. Example of a real spectrum N60. Black: The measured (instrumental) spectrum with visible artefacts. Grey: The recovered physical spectrum with almost full compensation of the artefacts. We can see correction of the detection efficiency and correction of the escape effect.

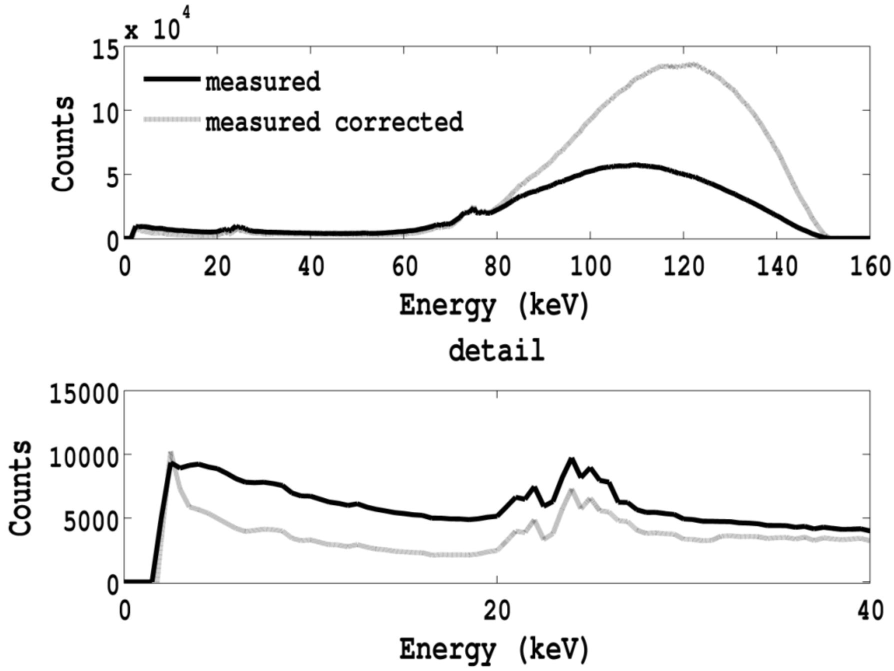

Figure 3. Example of a real spectrum N150. Black: The measured (instrumental) spectrum with artefacts. Grey: The recovered physical spectrum.

For quantitative analysis of the measured corrected spectra the following parameters were used:

E(max) – maximum energy in the measured corrected spectra. Precision of assessment is ±(1-2) channel width.

E(Imax) – energy corresponding to the maximal number of impulses in the measured corrected spectrum of bremsstrahlung radiation in the range (0, 160 keV).

E(mean) – mean energy in the measured corrected spectrum. $E(mean)=\Sigma(E_i*N_i)/\Sigma N_i$, where E_i is energy corresponding to the i-th channel and N_i is number of photons with the energy E_i.

FWHM – full width in the half of the maximum of bremsstrahlung in the measured corrected spectrum.

R – resolution; R=FWHM/E(mean).

N60	E(max) [keV]	E(Imax) [keV]	E(mean) [keV]	R [%]
Measured corrected	61	49.5	48.3	35.2
Norm ISO	60		48	36

Table 1A. Values of E(max), E(Imax), E(mean) and R derived from the measured corrected spectrum N60 compared with values cited from the norm (ISO 4037-1:1996).

N150	E(max) [keV]	E(Imax) [keV]	E(mean) [keV]	R [%]
Measured corrected	151.5	120	114.9	40.9
Norm ISO	150		118	37

Table 1B. Values of E(max), E(Imax), E(mean) and R derived from the measured corrected spectrum N150 compared with values cited from the norm (ISO 4037-1:1996).

The measured corrected spectra can be considered as being in a good agreement with the norm (ISO 4037-1:1996) where the mean energy is given with a precision of ±5% and the resolution with a precision of ±15%.

Conclusion:

The response matrix $R(E_j, E_k)$ of the Amptek CdTe XR-100T detector was constructed to enable numerical correction of the detection efficiency, escape effects and Compton scattering in the measured X-ray spectra with energies up to 160 keV. The incomplete charge collection was not considered. The matrix ***R*** was constructed analytically and can be used for spectra correction by a simple division of the vector ***B*** representing the measured spectrum by this matrix.

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