

IMPLEMENTATION OF PREISACH MODEL OF HYSTERESIS IN MATLAB

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Abstract

Preisach model can be applied only numerically. Theory of model shows that the suitable SW is the MATLAB, because of its many advantages. The key part of SW is simple and robust function for model processing. Also the MATLAB applications for model parameter finding and result exhibition are presented.

1. Introduction

Both in research and technical application many elements exist that exhibit hysteresis. Since their state depends on their history, their analysis is very complicated. It needs special instruments in comparison with usual nonlinear elements. Hysteresis must be accompanied by nonlinearity, of course. In the device design these elements are approximated by suitable models. The simplest model is the linear one that neglects both the hysteresis and nonlinearity. Models for small excitation consider nonlinearity, but neglects hysteresis. Model using harmonics of response can approximate the hysteresis effect but only for the case of harmonic excitation and relatively small amplitude. Only several sophisticated models consider hysteresis in full extent [2]. The most popular of them is the Preisach one [1].

Preisach model is known from the year 1935, when he was first presented [3]. Preisach approach is intuitive from the knowledge in magnetism, but later it was found that the model could be applied to all the areas of hysteresis.

Preisach model cannot be applied efficiently without the computer. Therefore, its application is growing with computer development. The second question is what SW should be used. We have found that the best SW for our purposes is MATLAB, because of its several advantages: very efficient processing of matrices, a lot of system functions, simple use of graphics, etc. The application of MATLAB for the hysteresis study of ferromagnetic materials is the contents of the paper.

2. Theory

Since the Preisach model is not well-known, in the theoretical part we describe it, using both the mathematical formulation and geometrical explanation. Furthermore, the determination of model parameters will be mentioned.

2.1 Preisach model

This model is based on elementary hysterons that exhibit rectangular hysteresis loop, Fig. 1. The momentum m of hysteron is set to 1 without loss of generality.

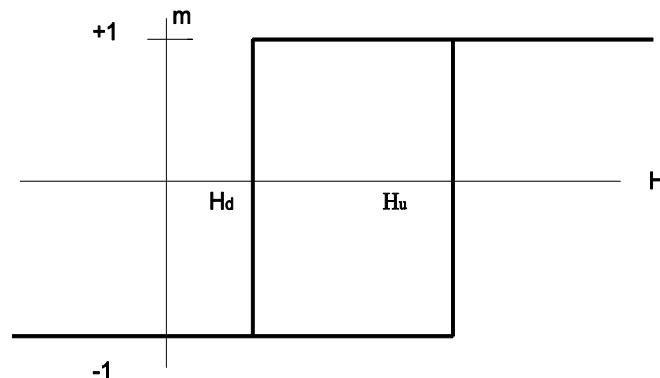


Figure 1. Hysteresis loop of elementary hysteron.

Let the external magnetic field strength H increases monotonically from very low negative value. The magnetic momentum of hysteron is -1. If the magnetic field strength exceeds the value of H_u (Fig. 1), the momentum is immediately switched to the positive value of +1. The momentum does not change when the external field further increases.

If the magnetic field increase is now stopped and the monotonically decrease follows, the magnetic momentum remains positive. However, if the value of magnetic strength decreases under the value H_d (Fig. 1), the dipole momentum immediately switches to negative value of -1 and this state does not change for further decrease of external magnetic field.

It is clear from the above description and Fig. 1 that the hysteron is fully given by only two parameters: magnetic field strengths H_u and H_d . The model works with hysterons whose parameters H_u and H_d change independently in wide areas. Only the condition $H_u \geq H_d$ is required. Of course, some parameter combinations are most probable, while the probability of others is low.

Total magnetic momentum is the integral over all the hysterons. Usually the magnetization M as the magnetic momentum of unit volume is used. Then the mathematical formulations of Preisach model is done by the formula

$$M(t) = \iint_{H_u \geq H_d} \varphi(H_u, H_d) \gamma(H_u, H_d) H(t) dH_u dH_d, \quad (1)$$

where $\varphi(H_u, H_d)$ is weighting function, γ is operator, which switches elementary hysterons into positive or negative saturation (or makes nothing). Symbol H defines the input (applied) magnetic field strength and magnetization M is the output from model. Since the exciting magnetic field varies with time t , the magnetization is also the function of time.

Weighting function $\varphi(H_u, H_d)$ is the characterization of magnetic material. It has physical meaning of 2D density of hysteron magnetic momentum. In the elementary area $dH_u dH_d$, centered at the point $[H_u, H_d]$, there is elementary magnetic momentum

$$dM = \varphi(H_u, H_d) dH_u dH_d \quad (2)$$

Continuous model (1) is not suitable for numerical calculation. In numerical implementation the integral is converted to the sum

$$M(t) = \sum_{i=1}^N \sum_{j=1}^i \varphi(H_u, H_d) \gamma(H_u, H_d) H(t) \Delta H_u \Delta H_d. \quad (3)$$

where the finite differences ΔH_u and ΔH_d are constant and usually $\Delta H_u = \Delta H_d$. In general, the smaller difference ΔH_u and ΔH_d , the better numerical accuracy can be expected.

2.2 Geometry of Preisach model

Geometrical explanation of Preisach model is based on systematically arranged hysterons in Fig. 2. The grid is very rough for numeric calculation. It was selected in order to see details and it is satisfactory for explanation. The parameter H_u increases from down to top, the parameter H_d increases from left to right. Because of condition $H_u \geq H_d$ only triangle, termed Preisach triangle, is used instead of expected square.

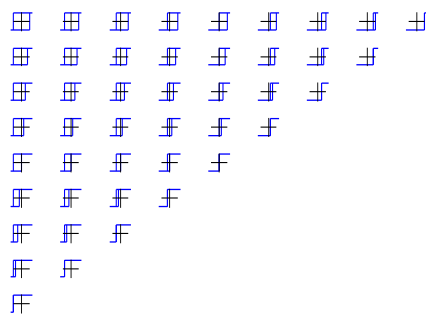


Figure 2. Hysterons of Preisach model

There are two important features of Preisach triangle. No hysteresis is on the main diagonal defined by condition $H_u = H_d$. Therefore main diagonal can be used for the modeling of nonlinearity. Comparing with nonlinearity, the hysteresis is much more complicated. It needs 2D structure, while the 1D structure is enough for nonlinearity.

The second feature is related to the secondary diagonal $H_u = -H_d$. The better term may be triangle height. The elementary loops are symmetrical on that diagonal. Since the experimental loops are usually symmetrical, we can expect the highest values of weighting function just on the secondary diagonal.

The function of Preisach model can be effectively explained from Preisach triangle. The initial state is negative saturation. It means that the exciting magnetic field strength H is negative and lower than the minimum of parameter H_u , which is value $-H_{u0}$ in Fig. 3. If the external field increases to some value H then all the hysterons with $H_u < H$ are switched up (Fig. 1). The situation is in Fig. 3. The external field strength H is given by horizontal line (moving up). Hysterons under this line are switched up, read area, while hysterons over the line remains switched down, blue area. The moving up of the horizontal line is finished after the maximum of external field strength is achieved. Then the decrease of external field follows.

If the input field strength decreases to the value H then all the hysterons with condition $H_d > H$ are switched down. The situation is in Fig. 4. The external field is given by the vertical line (moving to the left hand side). All the hysterons that are on the right hand part from this vertical line are switched down (blue color) in Fig. 4.

It should be stressed that the horizontal line corresponds to increasing field strength and it can move only from down to up. The decreasing field is connected with the vertical line that can move only from right hand side to the left hand side. The minimum of external field switches from vertical line to the horizontal one, while the maximum switches from the horizontal line to the vertical one.

The hysteresis in time domain, output waveform, can be recognised from the fact that the decreasing part of waveform is not a mirror image of increasing one. Nonlinearity preserves the mirror symmetry. The asymmetry follows simply from Preisach model and Fig. 3 and 4. The vertical line, if the external field decreases, switches other dipoles, or the same ones, but in another order.

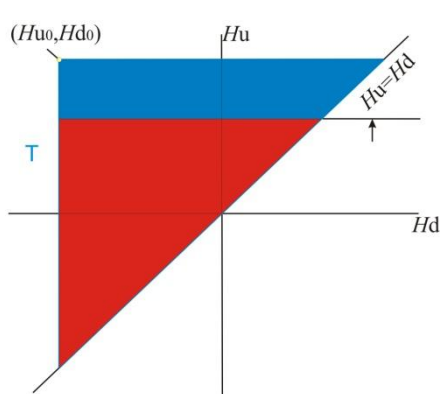


Figure 1. Moving the horizontal line from down to up for increasing field

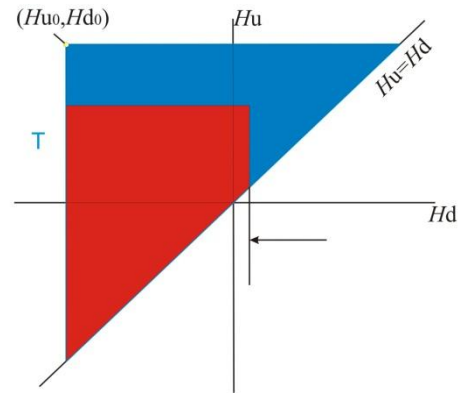


Figure 2. Decreasing field strength after maximum moves the vertical line from right to left hand side.

2.3 Weighting function

Weighting function is the key parameter of model, since it is the characteristics of studied magnetic material. It must be determined from measurements. There are two approaches for its determination. One approach is systematical, based directly on the measured magnetization from negative saturation by increasing amplitude; other approach is the estimation of the weighting function based on mathematical – physical considerations. This approach is termed for the purpose of this work as trials and errors approach.

Main problem of systematical approach is in the computation of weighting function from measured magnetization M , since there are two partial derivations [1],

$$\varphi(H_u, H_d) = \frac{1}{2} \frac{\partial^2 M(H_u, H_d)}{\partial H_u \partial H_d} \quad (4)$$

Derivation amplifies the noise. Really, direct application of numerical derivation gives bad result. We stress that this method is not applied in literature to our knowledge.

Trials and error approach is based on these mathematical – physical considerations. Weighting function has one single maximum at line $H_u = -H_d$, since the measured loop is symmetrical. The position of this maximum on the second diagonal can be derived from the measured loop width.

Furthermore the weighting function is symmetrical with respect the secondary diagonal, as it is shown in Ref. [1]. As suitable functions for these purposes we have found the functions used for probability density (Gauss, Cauchy and logarithmical- normal).

The comparison of methods is possible also from their information extent. The systematic method needs all curves from the minimum to maximum excitation. All material properties are included in this set of hysteresis loops. On the other hand, only one hysteresis loop for maximum excitation is necessary in the method of trials and errors. Therefore, this method is the only approximate one.

3. Experiment

The important conclusion from the Preisach model application is that the demagnetized state is not well-defined. The negative saturation is recommended as the starting point. The saturation may be also positive if a positive starting point of the model is used. The magnetizing current has increasing amplitude and negative bias as it is shown in Fig. 5, where is the time dependence of the primary current. The harmonic current excitation is preferred, but it needs to use low frequency, about 1 Hz. Otherwise, the apparatus is standard and the measurement is fully automated.

Magnetic flux density cannot be measured directly, but it is computed by numerical integration. This approach has big problem with offset voltage. At figure 6 we see the shift of the experimental loops due to the offset of measured cards and amplifiers. Corrections of offset are not simple, offset is not constant, and its evolution over time is unpredictable. As the best correction we have found the linear regression of secondary voltage before the start of excitation harmonic current. The linear values are then subtracted from measured voltage

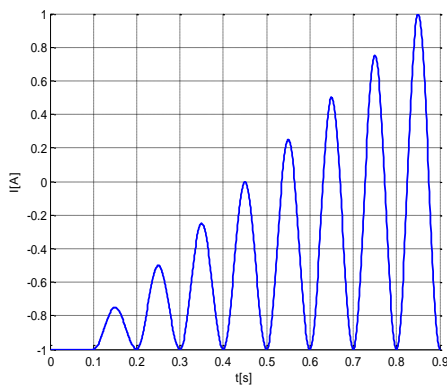


Figure 3. Current on the primary winding.

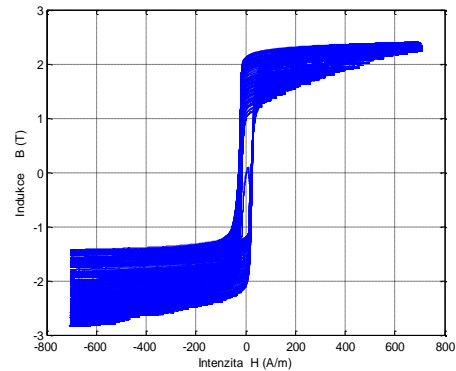


Figure 4. Problem with offset of input field.

4. Calculations

All the numerical calculations were made in MATLAB. The key part of the model is to use the discrete formula (3). Key operator γ can be represented subroutine that switches hysterons. Thanks to geometrical explanation, namely in Fig. 3 and 4, very simple MATLAB function can be created. The code follows. It is very simple, since logical indexing is used. Variable M and H represent state of hysterons in Preisach triangle and corresponding magnetic field strength, respectively. Important input parameters are u_{roven} that presents actual external magnetic field strength and logical variable $rust$

that specifies if the input field (*uroven*) increases or decreases. The variable *rust* switches between activity in Fig. 3 or Fig. 4. The output is actual values of hysterons in Preisach triangle.

```
function [ M ] = PM_podpr_polar(M, H, uroven, Mom, rust)

global PM
N=length(M);
if rust==1;           % pro rust polarizuj vse pod uroven kladne
    for ii=1:N
        M(H<uroven,1:ii)=Mom;
    end;
else
    for jj=1:N         % pro pokles polarizuj vse v pravo od urovne zaporne
        M(N:-1:jj,H>uroven)=-Mom;
    end;
end;
%M=M.*PM; % bere pouzije trojuhelnik
end
```

For the further use the weighting function must be applied to the elements in Preisach triangle. It is made by multiplication element by element of the weighting function. This matrix product is made very fast.

Other important application of MATLAB is the use of *fminsearch* function. It is used for the finding of the best parameters of weighting function by the method of trials and errors. The application of this function is complicated; therefore any details are not mentioned here.

5. Results

A lot of results were obtained with the application of Preisach model. Only small fraction of them is presented here. They are in two categories. The small category is made for education and the big part is for the use in scientific research.

5.1. Tutorial part

For the purpose of education the results can be only qualitative; therefore it is not important what weighting function is used. The constant value was set for simplicity. For better understanding of the model, its action is animated. The simple animation is shown if Fig. 7. It animates demagnetization by switching the current up and down with decreasing amplitude. Applied field is in the left part and the Preisach triangle is in right side. The situation roughly corresponds to continuous state in Fig. 4.

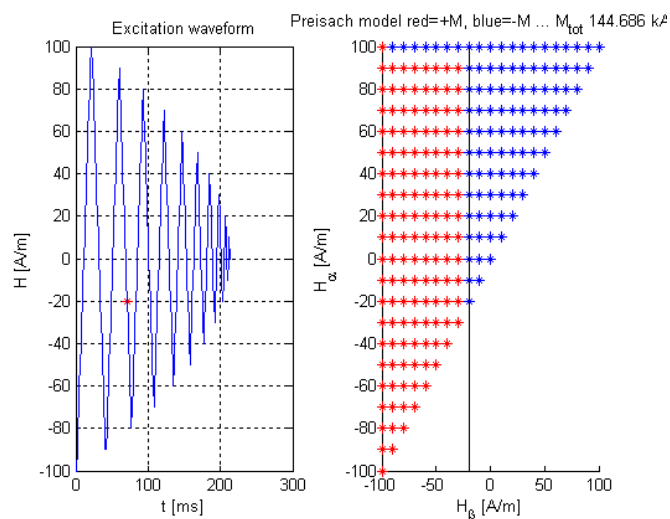


Figure 7. Simple animation of Preisach model.

The full animation is in Fig. 8. The excitation waveform is positioned left and up, the corresponding Preisach triangle is right and up, the time response is under the excitation and the hysteresis loop is under the triangle. The hysteresis loop can be relatively strange in general.

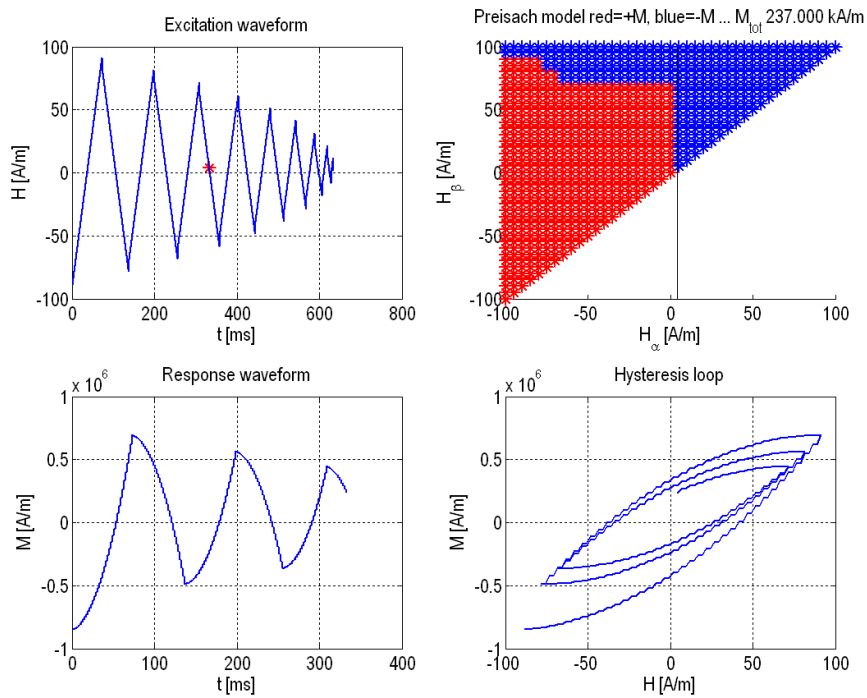


Figure 8. Full animation of Preisach model activity.

A final result of demagnetization is in Fig. 9. The total magnetization has not zero value, but small magnetic momentum remains. It is the reason, why in the experiment the initial state was not the demagnetized one, but the state of negative saturation.

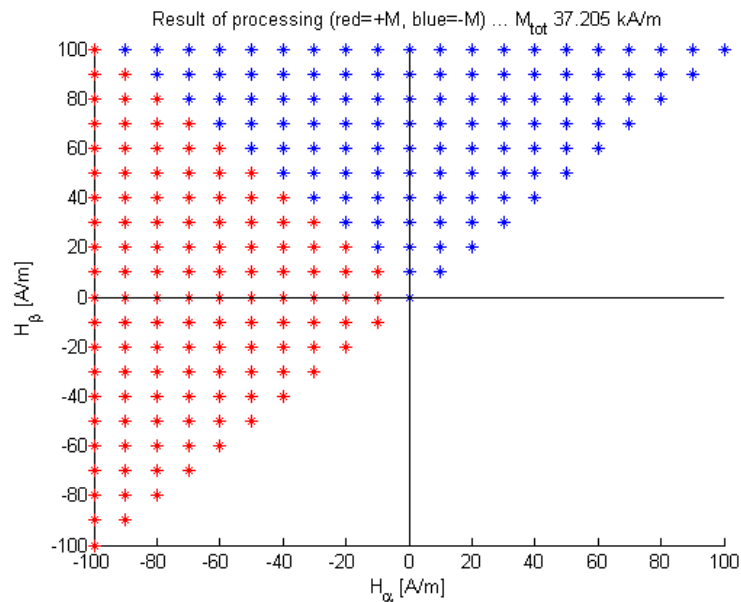


Figure 9. Result of demagnetization.

Well-known effect is the creation of small minor loops by harmonics in the excitation waveform. An example of minor loops is in Fig. 10 and 11. The exciting magnetic field in Fig. 10 contains the relatively large third harmonics. The resulting hysteresis loop in Fig. 11 contains two minor loops as we can expect. Furthermore, the position of minor loops depends on the phase angle of the third harmonics with respect to the first one. It can be simply modeled

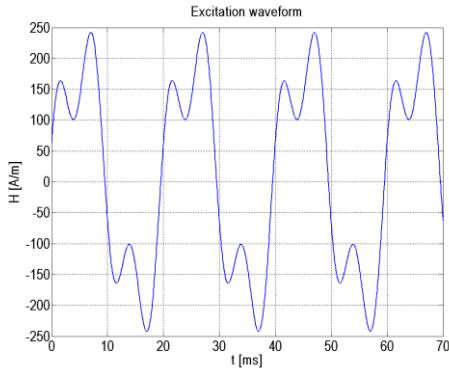


Figure 10. Input field waveform containing the third harmonics.

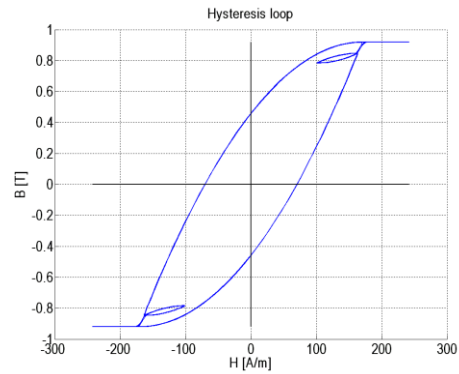


Figure 5. Modeling of minority loops.

5.2. Scientific area

In the research area the knowledge of the weighting function that respects the magnetic material is necessary. It is the key and very complicated step for the further technical applications of Presiach model. In the case of trials and errors method by the use of Cauchy distribution function and after optimizing its parameters by the fminsearch procedure the almost optimum weighting function was found. Its 2D shape is in Fig. 12. The relative sharp maximum is evident.

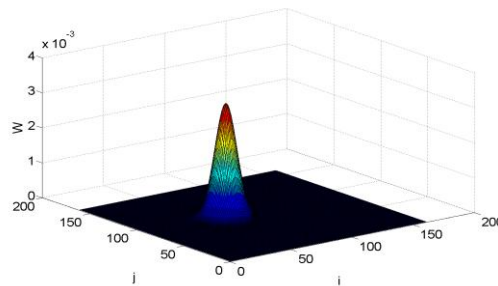


Figure 12. Weighting function as a result of optimization.

Then the weighting function is used for the determination of hysteresis loop. The comparison of calculated and measured hysteresis loops is in Fig. 13 and 14. The agreement is good, small deviations are in the area of knee (Fig. 14), but also in this area the accuracy is acceptable from the technical point of view.

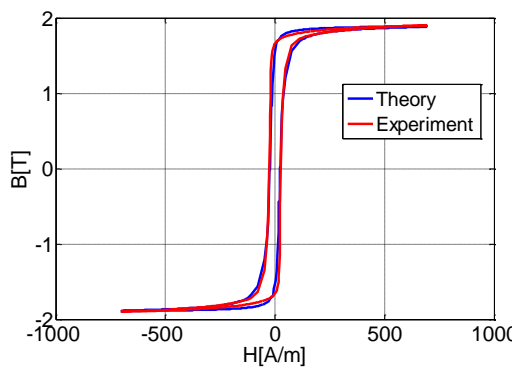


Figure 6 Comparison modeled and measured loops.

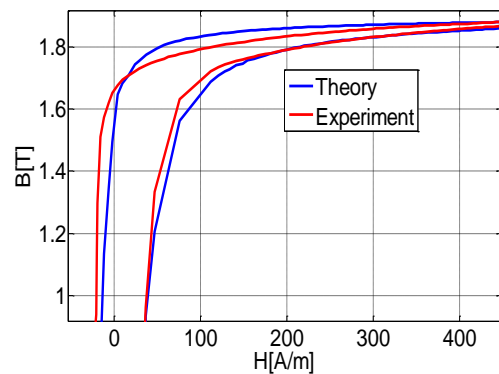


Figure 14. Details at knee of loops

Application of systematic method results to 2D field of response that is shown in the left part of Fig. 15. The measured data were processed in order to reduce unwanted effects. Then two partial derivations should be applied to the function according to formula (4). The numerical derivation results in the unacceptable results as it is presented in the right part of Fig. 15.

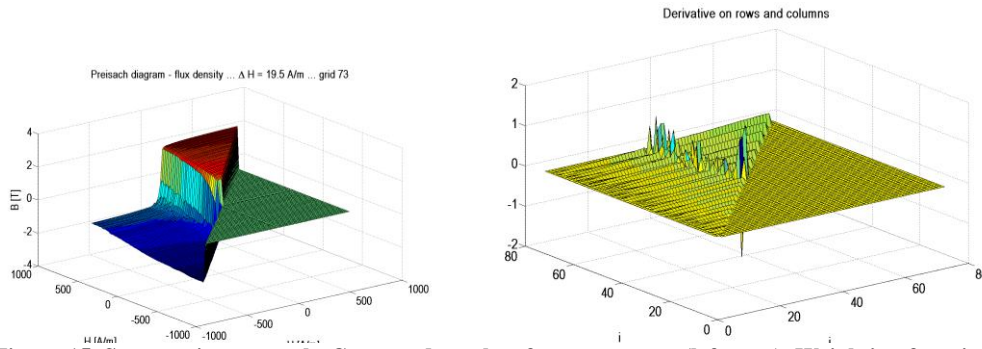


Figure 15. Systematic approach. Corrected results of measurement (left part). Weighting function obtained directly by application of partial derivations (right part).

The second attempt was the analytical approximation of the 2D function in Fig. 15, left part. Then the partial derivations in (4) can be made analytically. The function atan of three parameters was used for approximation. If the excitation is high, the approximation is good as it follows from Fig. 16. However, for the low excitation unacceptable approximation was obtained again.

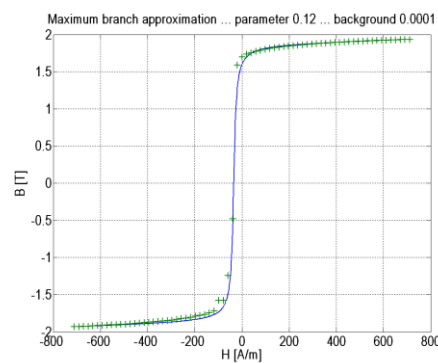


Figure 16. Approximation of the branch for high excitation.

6. Discussion

Application of Preisach model is possible only with a computer. Therefore, we should consider the robustness and speed of the script first of all. Model implementation is based on relatively simple algorithm, shown in part 3, Calculations. Thanks to its simplicity, the function is robust and no errors should appear in its use.

However, the speed is low, since the magnetic momentum of all the hysterons are calculated at every calling of the function. The speed can considerably increase, if only hysterons with changed momentum are found. It needs remembering of previous state and comparison. It leads to more complicated code and therefore the probability of errors is higher.

At present state we preferred reliability, since the speed was not critical in our application. In the future, if routine use will be required, the algorithm should be modified.

Very efficient was the application of the `fminsearch` procedure. But there are some limitations or danger, since the procedure often does not finish at optimum parameters or the result is unacceptable from other reasons. Therefore we run the procedure many times, sort the results according to sum of absolute deviations and graphically inspect the optimum or near optimum results. Then we get the reliable result in every case.

The method of trials and errors leads to weighting function that can be used in device analysis and design. It must be mentioned, however, that the model is valid near the excitation that was processed. If the excitation is strongly different, the check is necessary and, may be, corrected model parameters found.

The systematic method was not successful. One of reasons may be the experimental errors. Especially the curve drift, as in Fig. 6, can make problems. The drift is in the nature of the experiment and cannot be fully eliminated, but it can be considerably reduced. MATLAB has means for the efficient drift and noise reduction, but their inexact application can lead to data violation. Now the experiments are made with reduced drift and noise and realization of many near branches.

7. Conclusion

We have shown that MATLAB is a very suitable and efficient language for all the calculations connected with Preisach model starting with the control of the measurement and finishing with practical applications in the area of device analysis or design. Main reason is the very simple and fast matrix processing, many useful procedures, fast and simple graphics etc. The relative small speed can be in practical application compensated by parallel processing in many cases.

Acknowledgement

The work was supported by the Student Grant Competition of Technical University of Liberec.

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