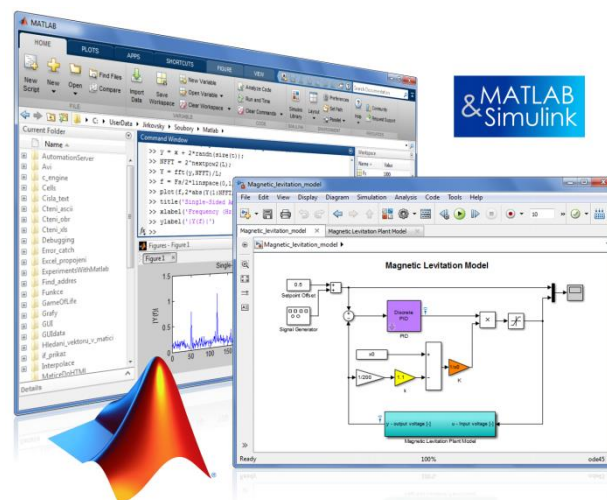


5.9.2019 Brno

TCC 2019

Řešení optimalizačních úloh v prostředí MATLAB



Jan Studnička
studnicka@humusoft.cz

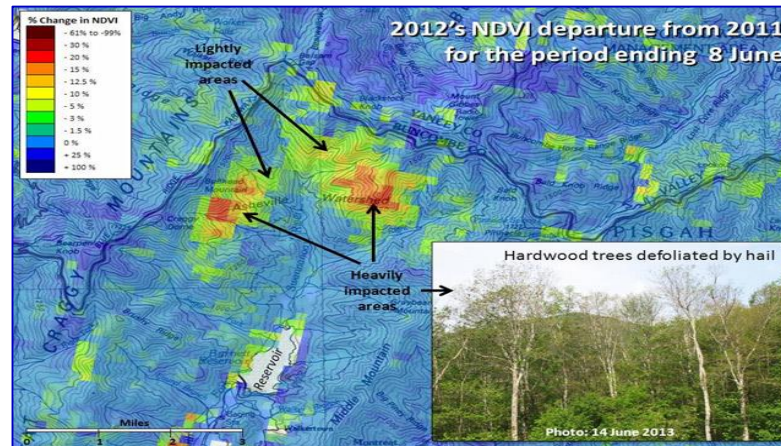
www.humusoft.cz
info@humusoft.cz

www.mathworks.com

Optimization is Used for Many Applications in Many Industries



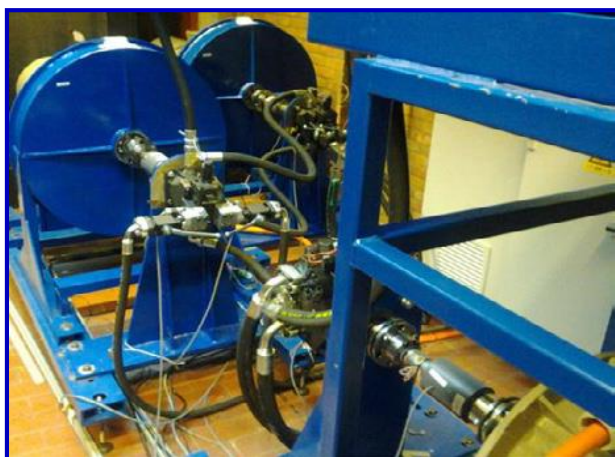
BuildingIQ: Manage HVAC



NASA: Identify forest disturbances



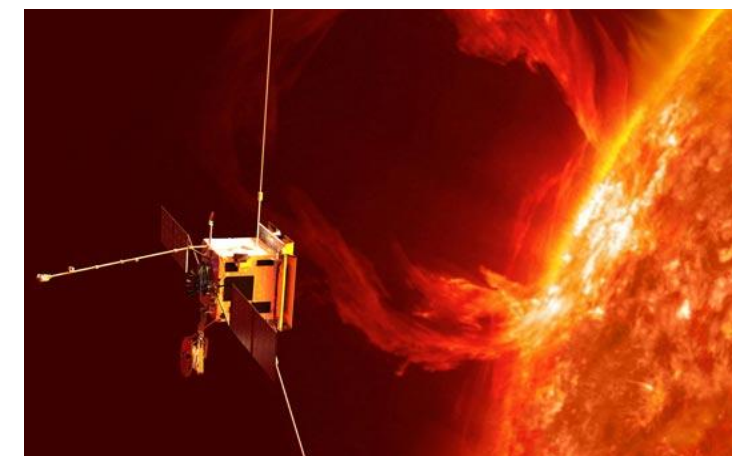
HKM: Plan steel production



FMTC: Estimate parameters

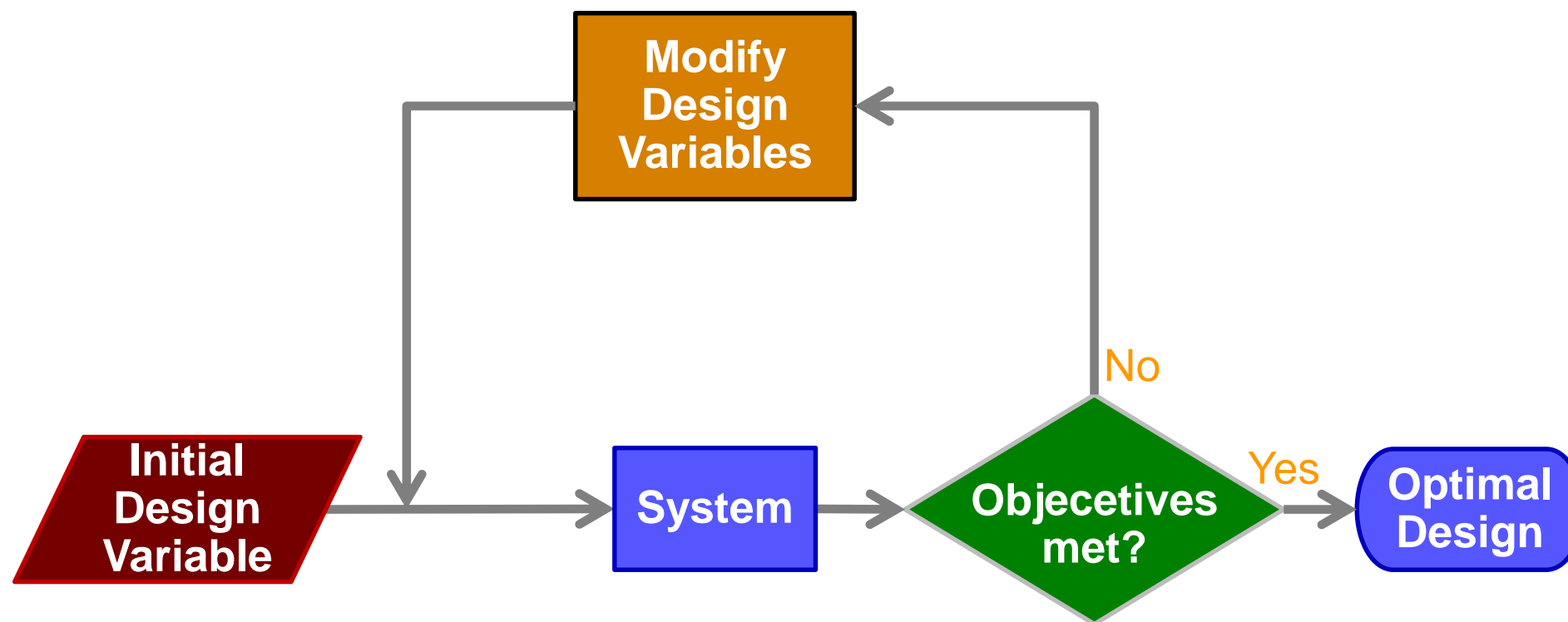


OTTO: Plan robot paths



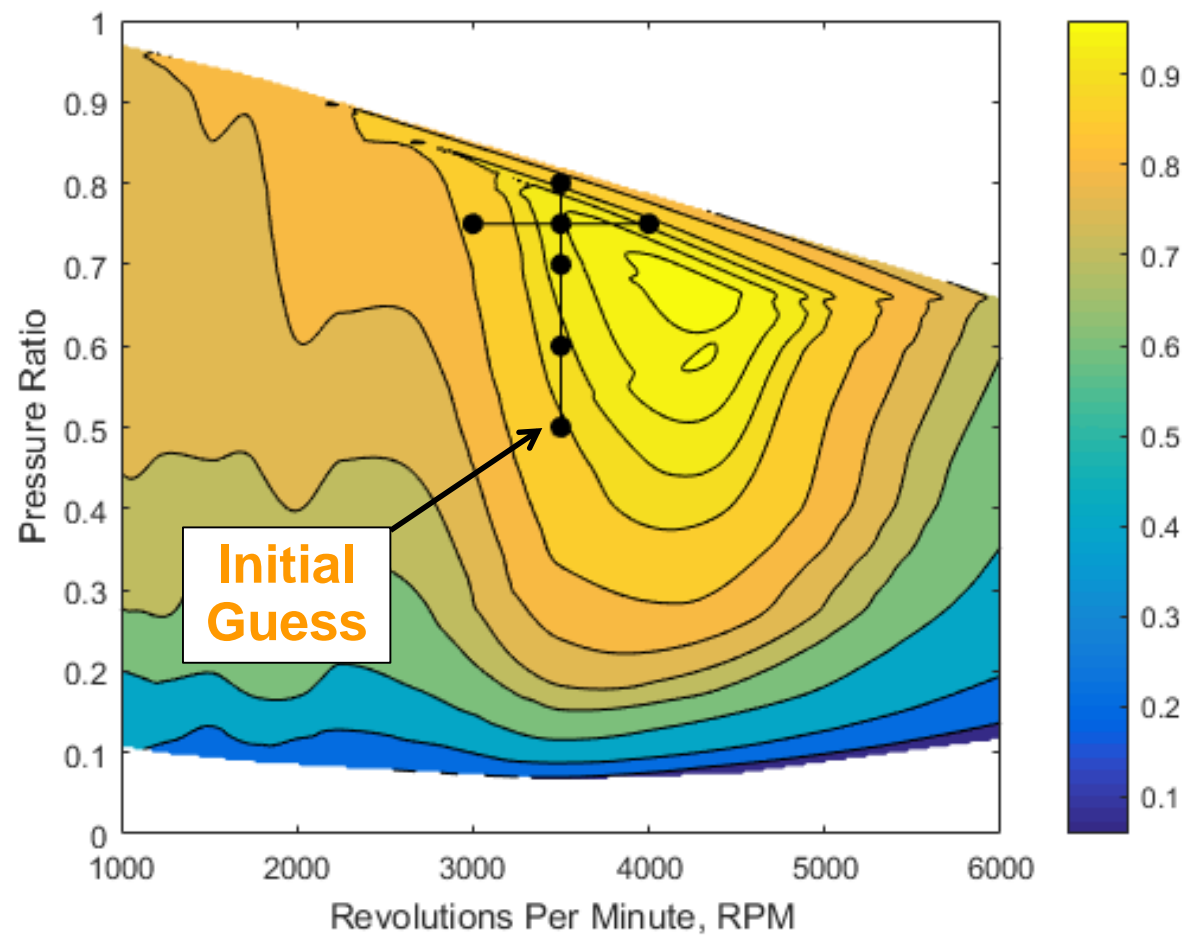
Tessella: Control Solar Orbiter

Design Process



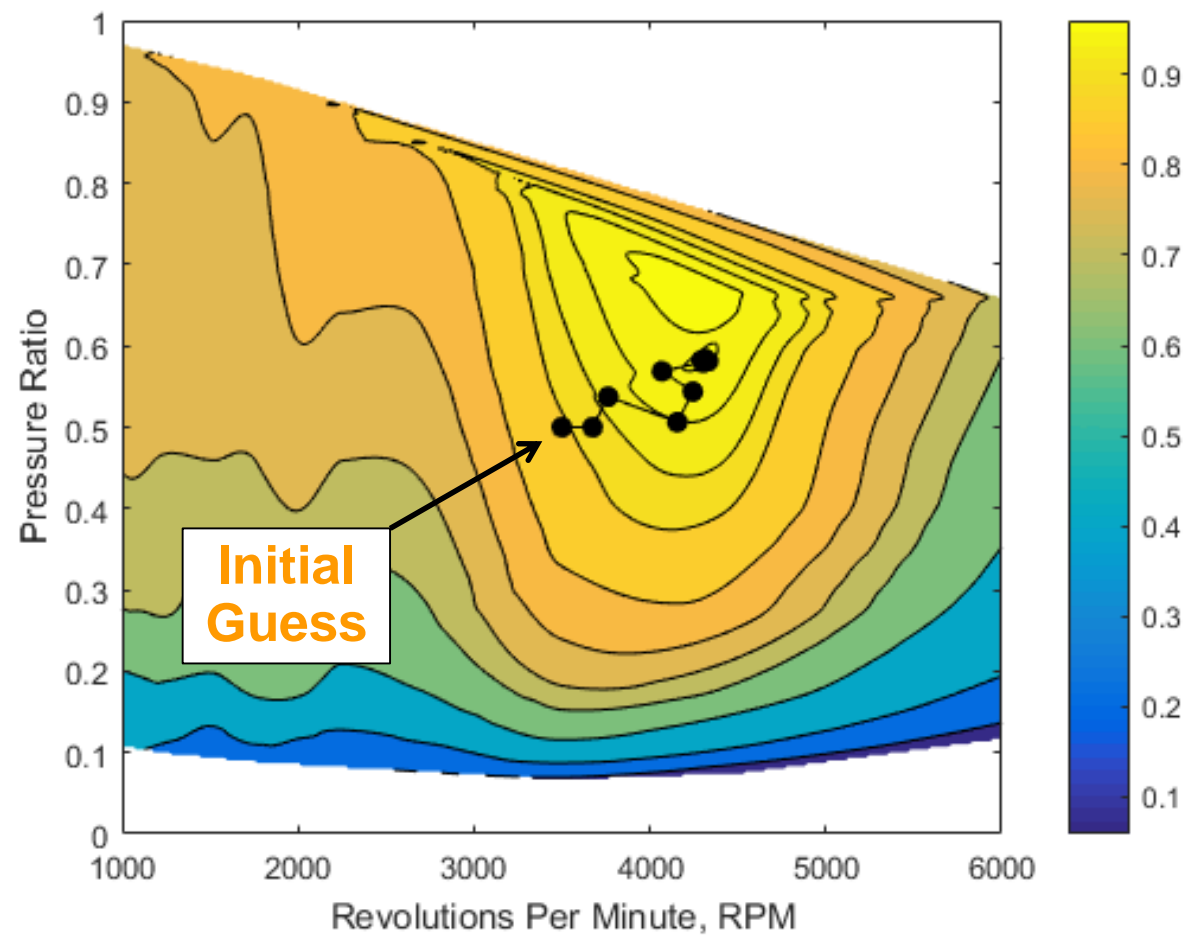
Why use Optimization?

Manually (trial-and-error or iteratively)



Why use Optimization?

Automatically (using **optimization** techniques)



Why use optimization?

Choose the optimal route to visit all MathWorks offices



28 offices

378 trips from one office to another

$1.0889e+28$ ways to assemble trips into a tour

Formulating an Optimization Problem: Index Fund Replication

Variables

What are my choices?

stocks

x_{ij}, y_j

x

Objective

What is my goal?

maximize similarity

maximize $\sum \sum c_{ij} x_{ij}$

f

Constraints

What restricts my choices?

one from each group

$$\sum_i x_{ij} = 1, j \in J_k$$

A, b, Aeq, beq, lb, ub

Modeling: Solver-Based Workflow

For linear, mixed-integer linear, quadratic and nonlinear programs

Choose solver

```
intlinprog    fmincon
```

Define linear constraints and objectives with matrix operations

```
f = zeros(nVars,1);  
ct = correlation';  
f(1:nx) = correlation(:);
```

```
numStocks = 20;  
Aeq = sparse(1,nVars);  
Aeq(1,nx+1:end) = 1;  
beq = numStocks;
```

Define nonlinear constraints and objectives with MATLAB functions

```
objfcn = @(x) log( 1 + 3*(x(2) - (x(1)^3 - x(1)))^2 + (x(1) - 4/3)^2 );
```

Call chosen solver

```
[xfinal, fval, exitflag, output] = ...  
    fmincon(objfcn, [-1; 2], [], [], [], [], lb, ub, [], options);
```


Formulating an Optimization Problem: Index Fund Replication

Variables

What are my choices?

stocks

x_{ij}, y_j



Objective

What is my goal?

maximize similarity

maximize $\sum \sum c_{ij} x_{ij}$



Constraints

What restricts my choices?

one from each group

$\sum_i x_{ij} = 1, j \in J_k$

~~$x_{11}, x_{21}, x_{31}, x_{41}, x_{51}$~~



Modeling: Problem-Based Workflow

Optimization Toolbox: optimization and least-squares problems

R2017b
R2018b
R2019a

Intuitive problem definition

```
fundprob.ObjectiveSense = 'maximize';  
fundprob.Objective = sum(sum(correlation.*x));
```

```
objective = 1/2*x'*Covariance*x;  
portprob2.Objective = objective;
```

Keep constraints as you'd write them

```
fundprob.Constraints.fundSize = sum(y) == numStocks;  
fundprob.Constraints.onlyOneStock = sum(x,2) == 1;
```

Automatic solver selection

```
[solution,objectiveValue,exitflag] = solve(fundprob);
```

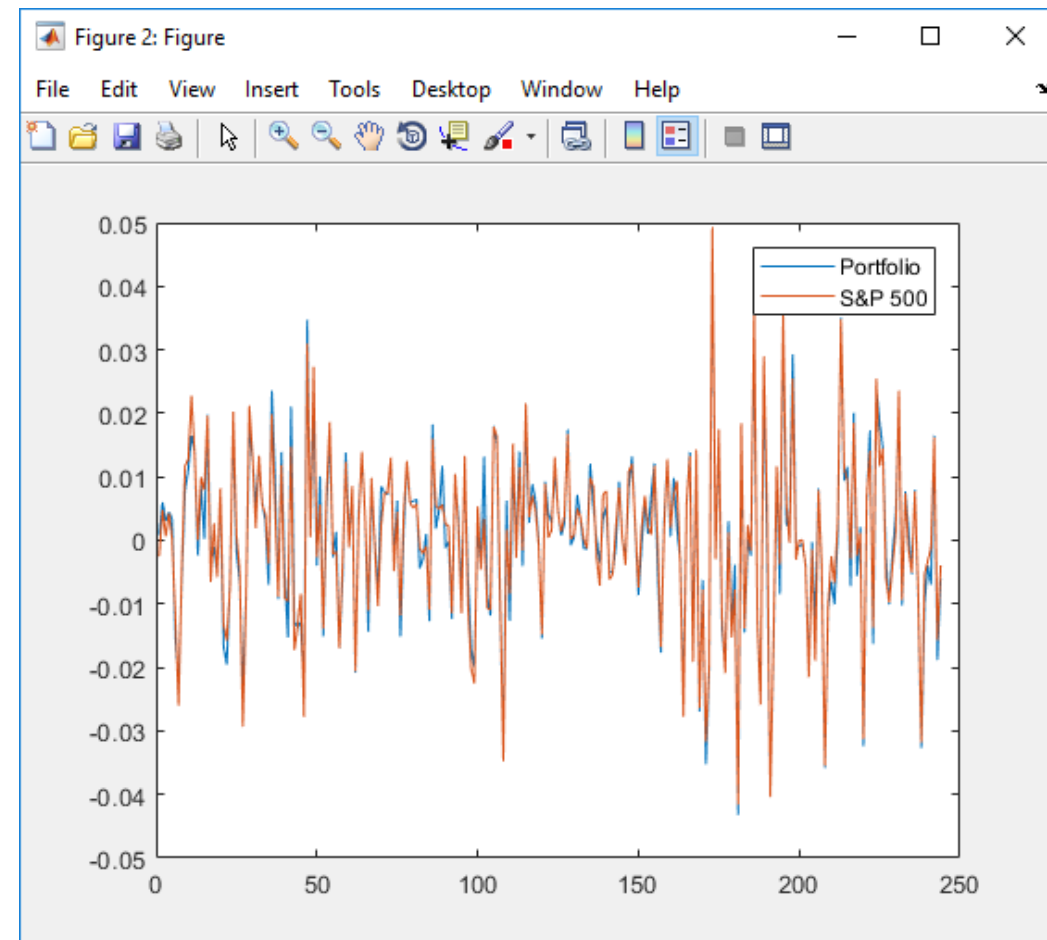
Demo: Index Fund Replication

Objective: Select a subset of stocks that match the returns of an index

Data: Closing prices for one year for stocks from the S&P 500

Approach:

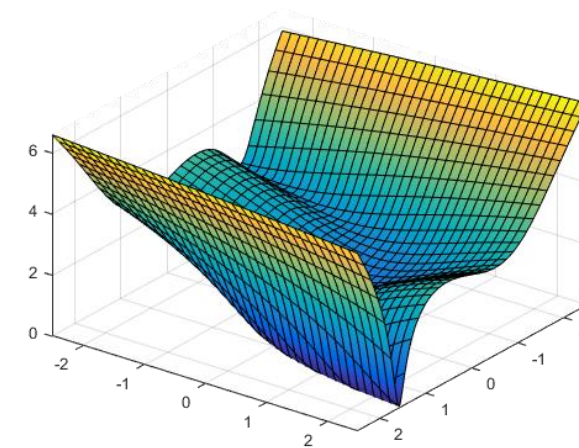
- Compute correlation to use as a measure of similarity
- Select stocks that maximize the similarity
- Each stock in the index is represented by a single stock in the subset



MathWorks Optimization Products

- **Optimization Toolbox**

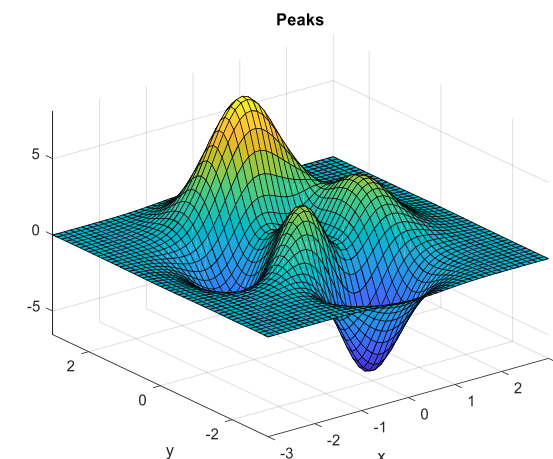
- Functions for finding parameters that **minimize or maximize objectives** while **satisfying constraints**



Objective with single minimum

- **Global Optimization Toolbox**

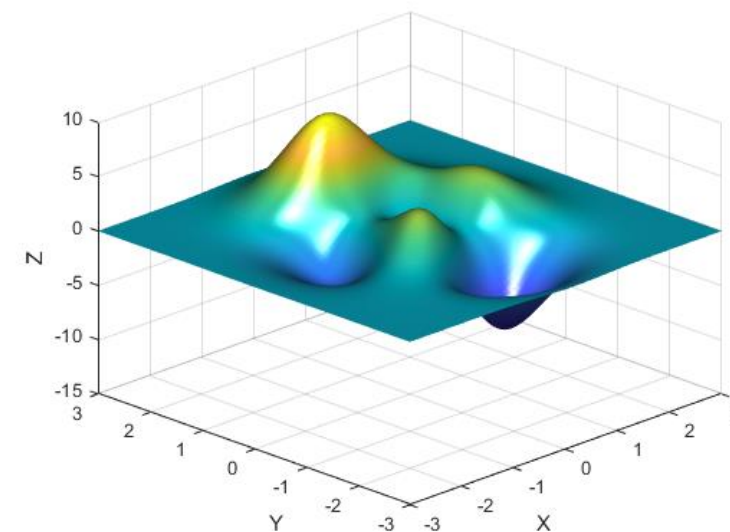
- Functions that **search for global solutions** to problems that contain **multiple maxima or minima** (*requires Optimization Toolbox*)



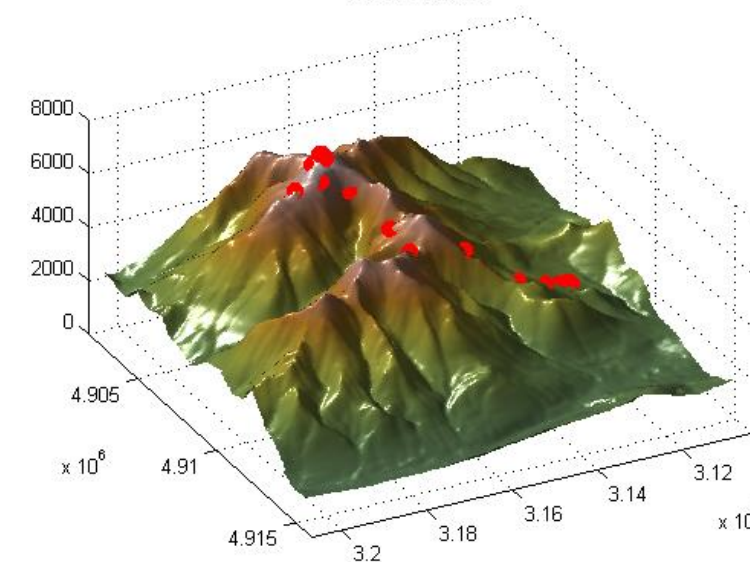
Objective with multiple minima

Optimization toolboxes support different problem types

	Optimization Toolbox	Global Optimization Toolbox
Faster	✓	
Large Problems	✓	
Better on: <ul style="list-style-type: none"> • Non-smooth • Noisy • Stochastic • Highly nonlinear 		✓
More “global”		✓
Custom data types		✓

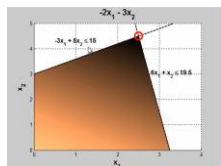


White mountains



Solving: Problem Types and Algorithms

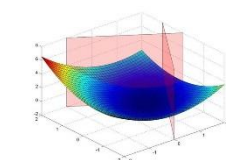
Optimization Toolbox *Global Optimization Toolbox*



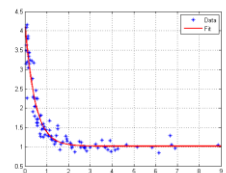
- **Linear programming**
 - Simplex and interior point



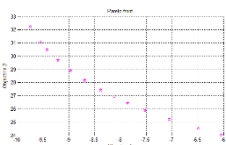
- **Mixed-integer linear programming**
 - Branch and cut



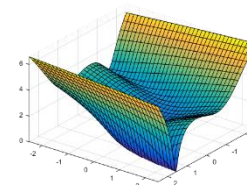
- **Quadratic programming**
 - Interior point and trust region



- **Least-squares and nonlinear equations**
 - Interior point, trust region, Levenberg-Marquardt

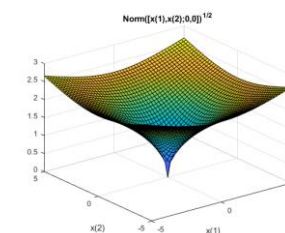


- **Multiobjective optimization**
 - Weighted and goal-attainment
 - *Genetic algorithm*
 - *Pattern (direct) search* **R2018b**

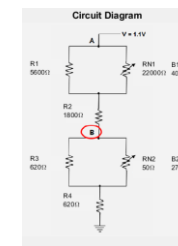


- **Nonlinear optimization**

- Interior point
- SQP
- Trust region
- Nelder-Mead simplex
- *MultiStart & GlobalSearch*
- *Pattern (direct) search*
- *Genetic algorithm*
- *Simulated annealing*
- *Particle swarm*
- **Surrogate optimization R2018b**

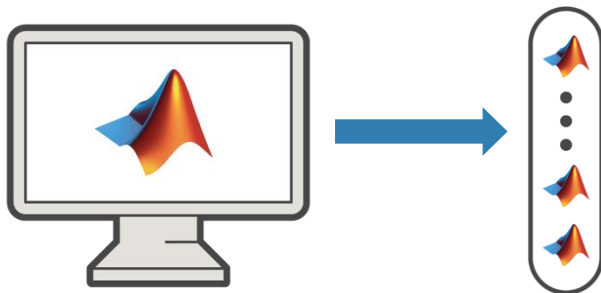


- **Mixed-integer nonlinear optimization**
 - *Genetic algorithm*

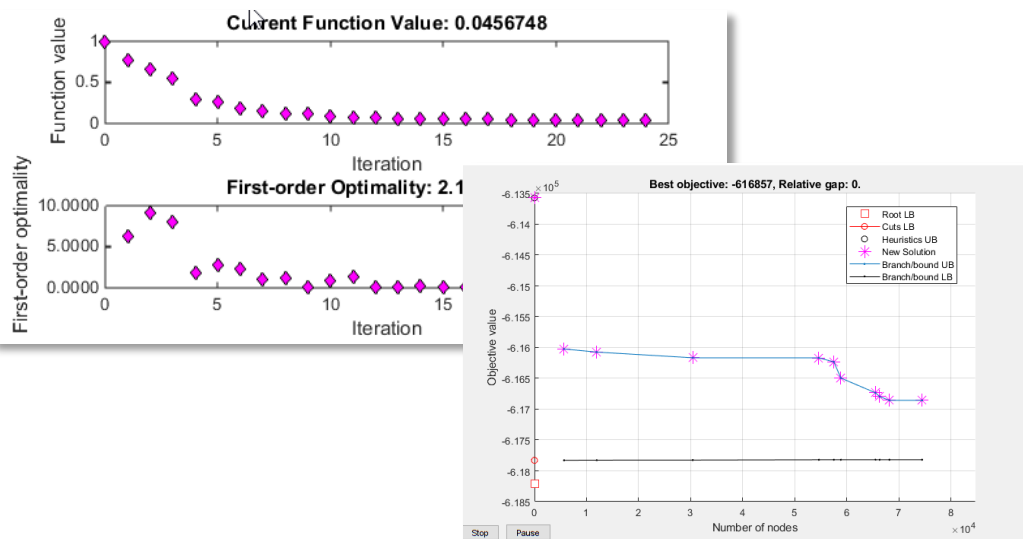


Solving: Features

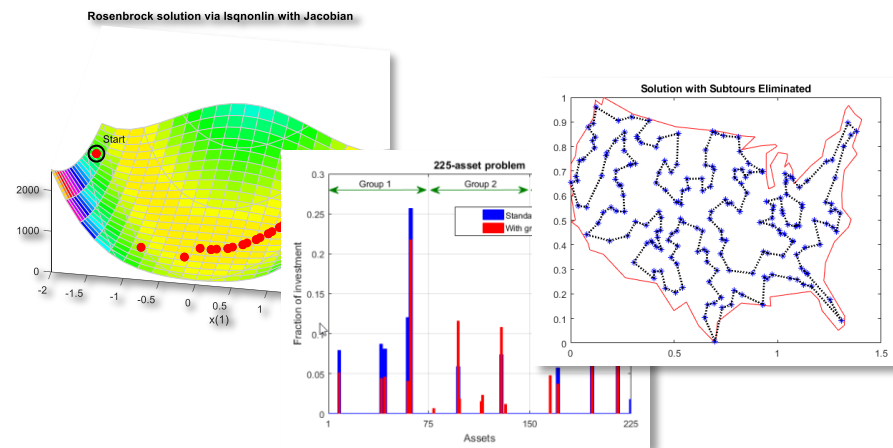
Parallel computing



Progress plots

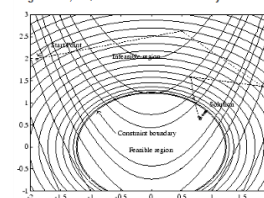


Examples and Tutorials



Algorithm Documentation

Figure 6-3, SQP Method on Nonlinearly Constrained Rosenbrock's Function



SQP Implementation

- The SQP implementation consists of three main stages, which are:
- Updating the Hessian Matrix
 - Quadratic Programming Solution
 - Initialization
 - Line Search and Merit Function

A subproblem formulation is defined as

$$\Theta(x, \lambda, s, \rho) = f(x) - \sum_{i=1}^m \lambda_i \log(s_i - c_i(x)) + \sum_{i=m+1}^m \lambda_i c_{eq_i}(x) + \frac{\rho}{2} \sum_{i=m+1}^m c_{eq_i}(x)^2,$$

where

- The components λ_i of the vector λ are nonnegative and are known as Lagrange multiplier estimates
- The elements s_i of the vector s are nonnegative shifts
- ρ is the positive penalty parameter.

The algorithm begins by using an initial value for the penalty parameter (`InitialPenalty`).

Optimization toolboxes support different problem types

Optimization Toolbox
Global Optimization Toolbox

Constraint Type	Objective Type					
	Linear	Quadratic	Least Squares	Smooth nonlinear	Nonsmooth	Multiobjective
None		quadprog	lsqcurvefit lsqnonlin	fminsearch fminunc	fminsearch <i>ga</i>	fgoalattain fminimax <i>paretosearch</i> <i>gamultiobj</i>
Bound		quadprog	lsqcurvefit lsqnonlin lsqnonneg lsqlin	fmincon	fminbnd <i>ga</i> <i>surrogateopt</i> <i>patternsearch</i> <i>particleswarm</i> <i>simulannealbnd</i>	fgoalattain fminimax <i>paretosearch</i> <i>gamultiobj</i>
Linear	linprog	quadprog	lsqlin	fmincon	<i>ga</i> <i>patternsearch</i>	fgoalattain fminimax <i>paretosearch</i> <i>gamultiobj</i>
General smooth	fmincon	fmincon	fmincon	fmincon	<i>ga</i> <i>patternsearch</i>	fgoalattain fminimax <i>paretosearch</i> <i>gamultiobj</i>
General nonsmooth	<i>ga</i> <i>patternsearch</i>	<i>ga</i> <i>patternsearch</i>	<i>ga</i> <i>patternsearch</i>	<i>ga</i> <i>patternsearch</i>	<i>ga</i> <i>patternsearch</i>	<i>paretosearch</i> <i>gamultiobj</i>
Discrete	intlinprog				<i>ga</i>	

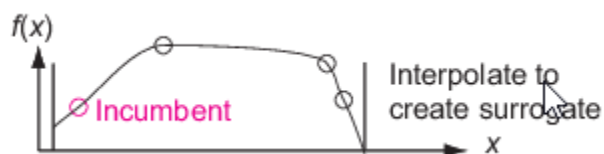
Solving: Recent Enhancements

- **Linear programming**
 - Dual simplex **more robust and 4x faster** than legacy interior point
 - New interior point **more robust and 3x faster** and than legacy interior point
- **Mixed-integer linear programming**
 - More **robust**
 - New **feasible point heuristics** and **branching methods**
- **Quadratic programming and linear least-squares**
 - New **interior point** algorithms
- **Nonlinear optimization**
 - **Faster SQP** algorithm
 - More algorithms use **parallelism**

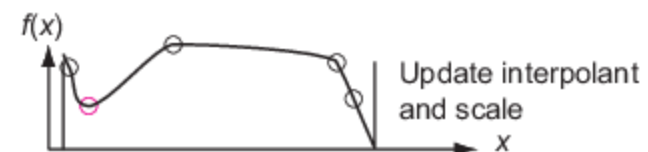
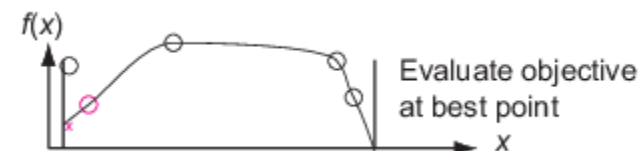
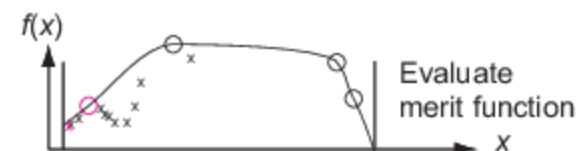
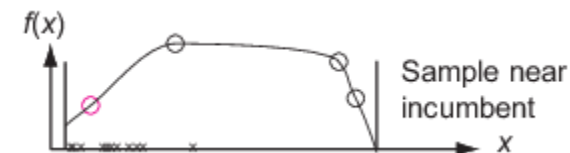
Solving: Surrogate Optimization

- Use on optimization problems that are expensive to evaluate
 - Simulations, differential equations
 - Uses fewer function evaluations than other Global Optimization solvers
 - Does not rely on gradients: works on smooth and nonsmooth problems

Construct Surrogate



Search for Minimum



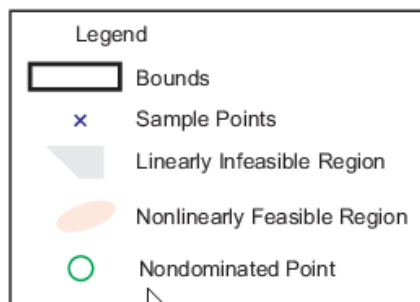
Reset



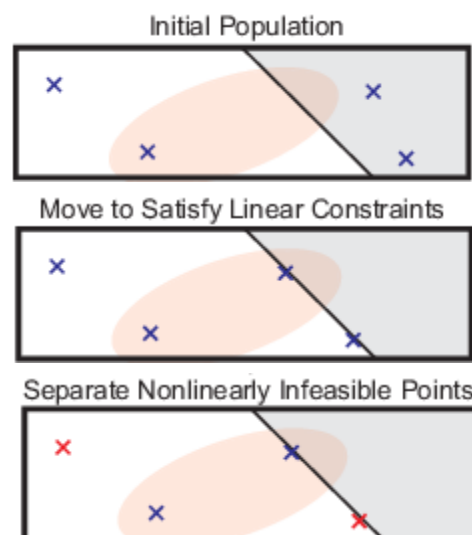
Solving: Multiobjective Pattern Search

R2018b
[video](#)

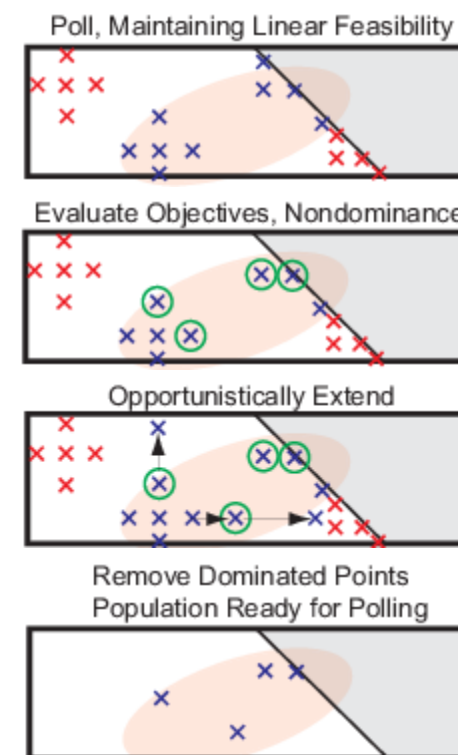
- Identify a Pareto set using a pattern search algorithm
- Use fewer function evaluations than multiobjective genetic algorithm



Initialize



Poll

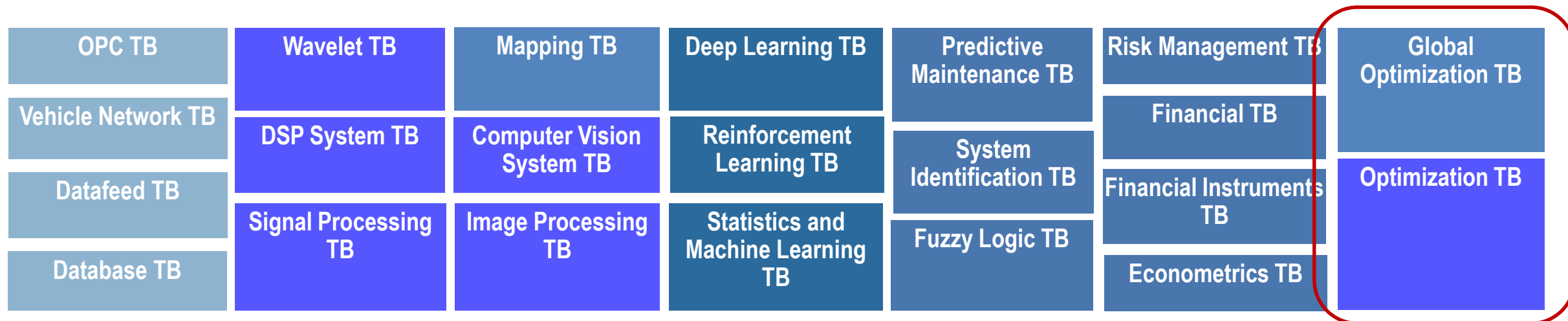


Iterate

Combining Predictive and Prescriptive Analytics

- | | | |
|--|---|--|
| Forecast electricity demand... | → | Decide which generating plants to use (<i>unit commitment problem</i>) |
| Predict demand for airline seats... | → | Set prices to maximize revenue (<i>revenue management</i>) |
| Forecast demand at stores... | → | Site warehouses to meet demand (<i>facility location</i>) |
| Predict returns and risks... | → | Allocate assets to balance return and risk (<i>portfolio optimization</i>) |
| Build a predictive model of response to marketing offers... | → | Decide which offers go to which customers to maximize response (<i>campaign optimization</i>) |

MATLAB is a Platform for Optimization



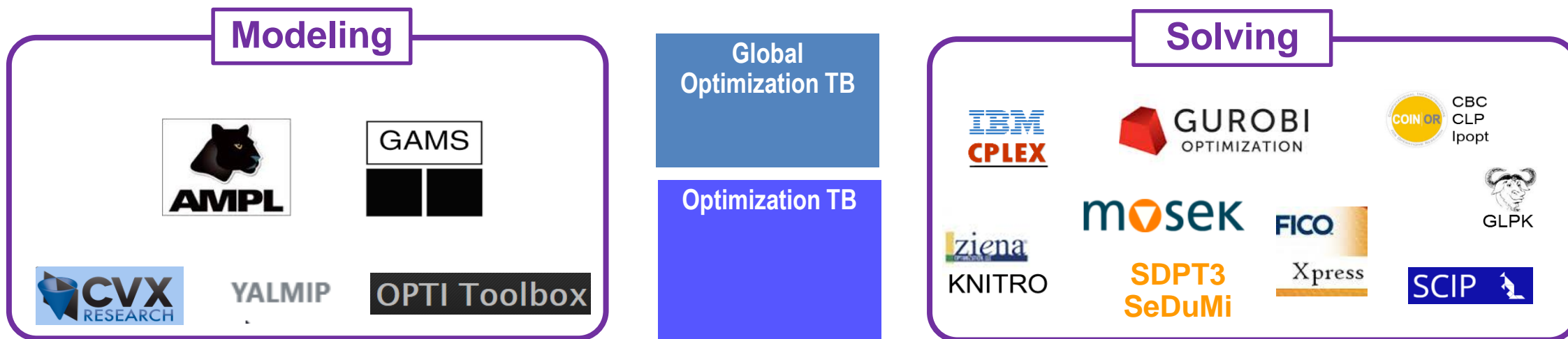
Parallel Computing TB and MATLAB Distributed Computing Server

MATLAB®



MATLAB is a Platform for Optimization

MATLAB plus toolboxes and 3rd party modeling languages and solvers



Parallel Computing TB and MATLAB Distributed Computing Server

MATLAB®



Develop Models

Optimization in MathWorks Toolboxes

- Financial Toolbox
- Financial Instruments Toolbox
- Econometrics Toolbox
- Risk Management Toolbox
- Simulink Design Optimization
- Model Based Calibration Toolbox
- Model Predictive Control Toolbox
- Robotics System Toolbox
- SimBiology

Hyperparameter Optimization

- **Machine Learning / Deep Learning**

- **design variables**

- internal parameters of a classifier or regression function

- **objective**

- minimize loss
 - time-consuming – requires training in each step

- **Bayesian Optimization**

- **well-suited to optimizing hyperparameters**

- low dimension, expensive objective, global solution

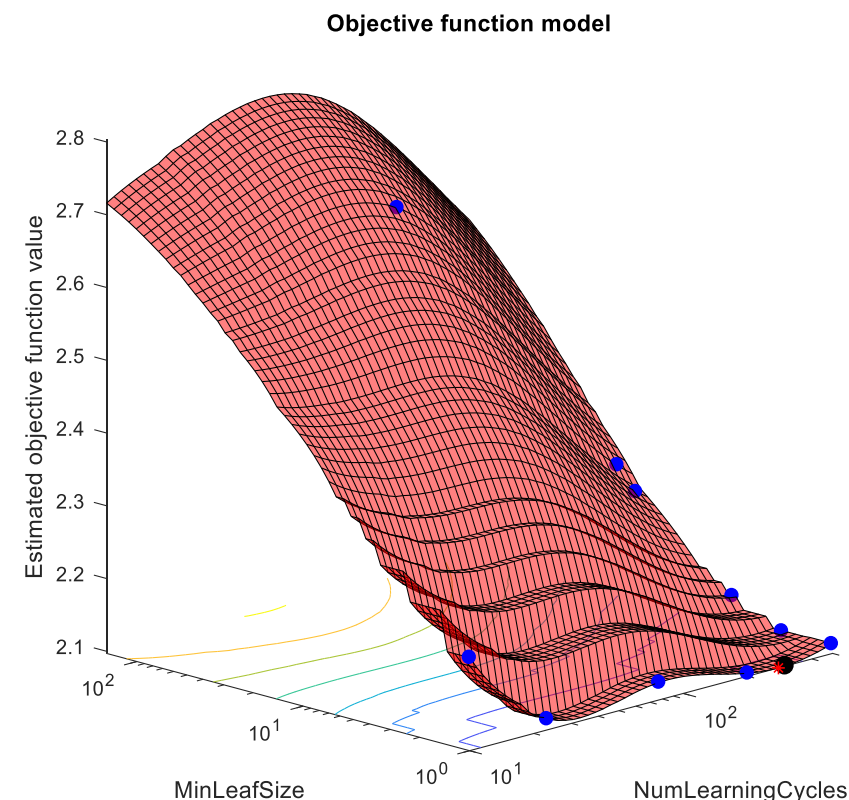
- **automatic**

- ‘*OptimizeHyperparameters*’ Name-Value pair

- **customized**

- *bayesopt* function

- **Statistics and Machine Learning Toolbox**



User Stories

BuildingIQ Develops Proactive Algorithms for HVAC Energy Optimization in Large-Scale Buildings

Challenge

Develop a real-time system to minimize HVAC energy costs in large-scale commercial buildings via proactive, predictive optimization

Solution

Use MATLAB to analyze and visualize big data sets, implement advanced optimization algorithms, and run the algorithms in a production cloud environment

Results

- Gigabytes of data analyzed and visualized
- Algorithm development speed increased tenfold
- Best algorithmic approaches quickly identified



Large-scale commercial buildings can reduce energy costs by 10–25% with BuildingIQ's energy optimization system.

“MATLAB has helped accelerate our R&D and deployment with its robust numerical algorithms, extensive visualization and analytics tools, reliable optimization routines, support for object-oriented programming, and ability to run in the cloud with our production Java applications.”

- Borislav Savkovic, Building IQ

HKM Optimizes Just-in-Time Steel Manufacturing Schedule

Challenge

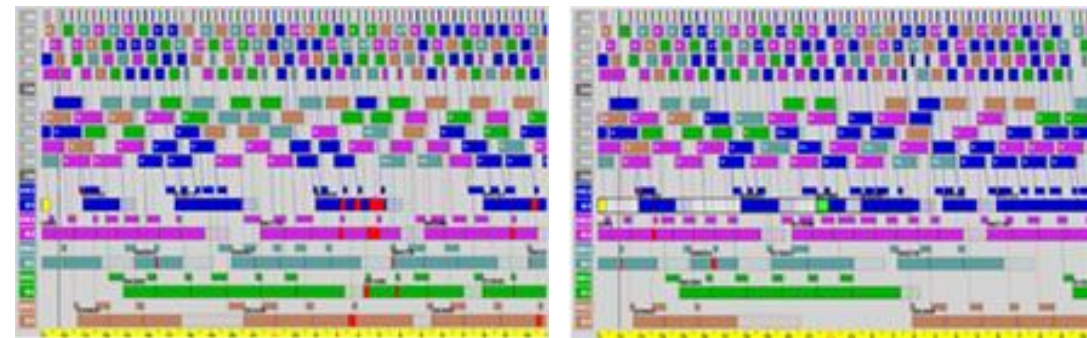
Optimize a steel production process to enable consistent, just-in-time delivery

Solution

Use MATLAB and Global Optimization Toolbox to maximize throughput of more than 5 million tonnes of steel annually

Results

- Algorithm development accelerated by a factor of 10
- Optimization time cut from 1 hour to 5 minutes
- Customer satisfaction increased



Manually reviewed plant schedule (left) and plant schedule automatically optimized with MATLAB genetic algorithms (right). The optimized schedule minimizes schedule conflicts (in red), meets delivery dates, and achieves the target utilization rate.

“C++, Java, or third-party optimization solutions would have required us to spend significantly more time in development or to simplify our constraints. Only MATLAB provided the flexibility, scalability, development speed, and level of optimization that we required.”

- Alexey Nagaytsev, Hüttenwerke Krupp Mannesmann

FMTC Designs and Optimizes a Hybrid Hydrostatic Drivetrain with Model-Based Design

Challenge

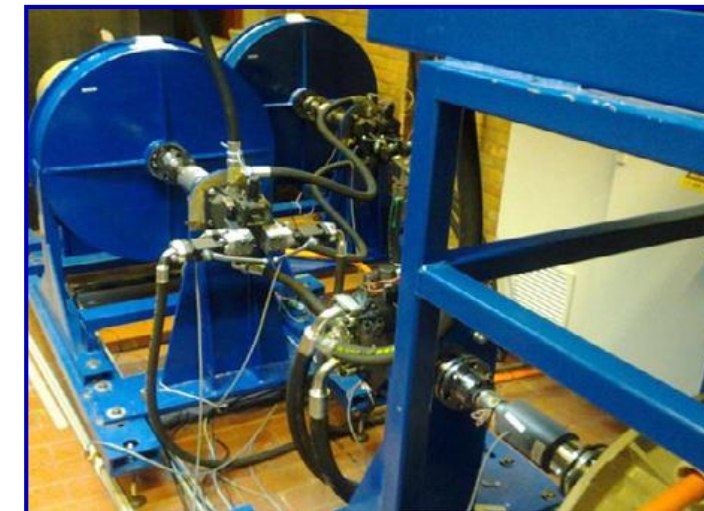
Evaluate energy-storage technologies for a hybrid hydrostatic drivetrain, and identify the most cost-effective alternative

Solution

Use Simulink, Simscape, and SimHydraulics to model, simulate, and optimize capacitor and hydraulic accumulator energy-storage components, hybrid hydrostatic drivetrains, and controllers

Results

- Fuel use reduced by 25%
- Analysis time cut by 75%
- Total cost of ownership reduced by 15%



The hybrid hydrostatic drivetrain setup.

“Model-Based Design supports a systematic approach to the design of drivetrains and other complex mechatronics systems. Detailed analysis of design alternatives based on the simulation of dynamic physical models and optimal controllers enabled us to make informed decisions early in the design phase.”

- Kristof Berx, FMTC

STIWA Increases Total Production Output of Automation Machinery

Challenge

Apply sophisticated mathematical methods to optimize automation machinery and increase total production output

Solution

Use AMS ZPoint-CI to collect large production data sets in near real time and use MATLAB to analyze the data and identify optimal trajectories

Results

- Development time reduced by one year or more
- Coding errors eliminated
- 80% model reuse achieved



STIWA's shopfloor management system, based on MATLAB, AMS ZPoint-CI, and AMS Analysis-CI.

“Our shopfloor management system AMS ZPoint-CI collects a huge amount of machine, process, and product data 24 hours a day. By analyzing this data immediately in MATLAB and AMS Analysis-CI we have achieved a tenfold increase in precision, a 30% reduction in total cycle time, and a significant increase in production output.”

- Alexander Meisinger, STIWA

Clearpath Robotics Accelerates Algorithm Development for Industrial Robots

Challenge

Shorten development times for laser-based perception, computer vision, fleet management, and control algorithms used in industrial robots

Solution

Use MATLAB to analyze and visualize ROS data, prototype algorithms, and apply the latest advances in robotics research

Results

- Data analysis time cut by up to 50%
- Customer communication improved
- Cutting-edge SDV algorithms quickly incorporated



An OTTO self-driving vehicle from Clearpath Robotics.

“ROS is good for robotics research and development, but not for data analysis. MATLAB, on the other hand, is not only a data analysis tool, it’s a data visualization and hardware interface tool as well, so it’s an excellent complement to ROS in many ways.”
- Iliia Baranov, Clearpath Robotics

Tessella Designs Attitude and Orbit Control Algorithms for Solar Orbiter Spacecraft Using Model-Based Design

Challenge

Design algorithms for the attitude and orbit control subsystem for the Solar Orbiter spacecraft capable of maintaining pointing stability to within a few tenths of an arcsecond

Solution

Use Model-Based Design with MATLAB and Simulink to model spacecraft sensors, actuators, and control algorithms; run simulations to optimize and tune the algorithms; and guide the creation of a detailed software specification

Results

- ECSS compliance demonstrated
- Complex analysis completed on schedule
- Models reused on follow-on projects, cutting design effort by up to 80%



Artist's rendition of the Solar Orbiter.

“We saw the benefits of Model-Based Design on several previous projects. On this project, MATLAB and Simulink enabled us to create a detailed specification that minimized deviation between the prototype algorithms we developed, tuned, and tested in Simulink and the final software implementation.”

- Andrew Pollard, Tessella

NASA Develops Early Warning System for Detecting Forest Disturbances

Challenge

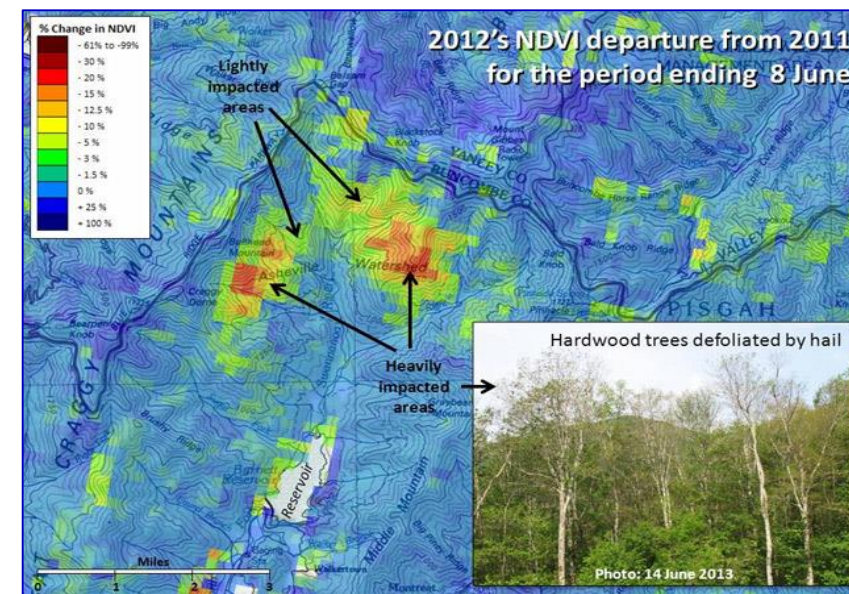
Develop a system that uses satellite imagery to quickly detect forest disturbance threats from insects, drought, storms, blights, wildfires, and other events

Solution

Use MATLAB to process multispectral satellite images, construct multidimensional time-series data baselines, and analyze terabytes of data to help detect regionally evident forest disturbances

Results

- New ideas implemented and tested in hours
- Years of development time saved
- Reusable production software delivered to growing user community



U.S. Forest Change Assessment Viewer map showing damage to the Asheville, North Carolina watershed following a 2012 hail storm. Image courtesy ForWarn.

“Soon after ForWarn moved into production, it detected previously unnoticed hail damage that posed a threat to a watershed. We would not have been able to do this work as efficiently without MATLAB.”

– Duane Armstrong, NASA Stennis Space Center