Educational electronic course "Theory of the electromagnetic field" on the basis of the program complex COMSOL Multiphysics

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Abstract

In the submitted paper the educational electronic course «Theory of the electromagnetic field» which uses the basic tools of the program complex COMSOL Multiphysics is considered. A basis of this electronic course is the standard course of lectures «Theory of the electromagnetic field» usually presented to the students who are studying the electrotechnical specialities in the Russian technical universities. The contents of the course includes five sections (General problems of the theory of the electromagnetic field (EMF), the Electrostatic field, the Electric field of a direct current in conducting medium, the Magnetostatic field and the Variable harmonic EMF). We have not include in structure of the course the section "The EMF in moving mediums" because this section usually does not a part of the standard student's course for these specialities.

A great number of exercises and demonstration examples which need to show, how with help of COMSOL Multiphysics it is possible to solve the various problems of the EMF theory, is entered into the course. Among the used tools of COMSOL Multiphysics there are the COMSOL Script, the PDE Modes, the Electromagnetics applications modes of COMSOL Multiphysics nucleus and the Module of electromagnetism. Examples and exercises will allow students to acquire better knowledge from the submitted materials and to receive very important skills of work with modern toolkits for modeling and solving various problems of EMF.

It is supposed that the hypertext of the course and the version of program complex COMSOL Multiphysics will arrange on one CDROM. Thus version of COMSOL Multiphysics will be specially adapted under used demonstration examples and exercises. For modeling of each exercise or example, the necessary COMSOL Multiphysics tools will be call from the text of the course. The students will have opportunity to change some conditions of the investigated problem, to enter new values of parameters and to use graphic and other means of the program complex for output, observation and registrations of the results of the solution.

1. Introduction

Now in the world, a rapid development of the educational electronic means is observed. One of approaches in the development of these means is connected with the new possibilities of modern technologies that allow to arrange on the uniform data medium (for example, CDROM) not only a teaching material but also the powerful software. This software makes possible to realize the modern representation of this material to the user and thus to apply the high technologies in educational processes. In the present paper the similar approach is applied to the creation of the lecture course where the main mathematical tools are the partial differential equations.

As such course the course "The theory of electromagnetic field" is chosen. In this course all exercises and the demonstration examples explaining the separate positions of the course are carried out by means of the COMSOL Multiphysics software complex distributed by COMSOL AB from Sweden.

The mathematical problems of the course includes the solutions of the different partial differential equations with various boundary and initial conditions. It demands to use the powerful software that can model the given class of problems in real time with application of developed systems of postprocessing and visualization of results. It is necessary to note, that among such courses there are the majority of the general courses of physics, chemistry, biology and other fundamental sciences, and also a number of specialized courses on separate theoretical and applied problems. With reference to a direction connected with EMF, the examples of similar specialized courses can serve: the theoretical foundations of electrical engineering, the theory of electrical machines and mechanisms, the problems of generation and distribution of electromagnetic waves, the microwave theory and applications and others.

The COMSOL Multiphysics software complex are a good basis for a developed course because it possess the big universality in relation to classes of the solved problems, good functional properties and wide modern opportunities on representation and processing of information. A fact that COMSOL AB has stated interest in joint development of this electronic course on the basis of COMSOL Multiphysics is important also. The realization of the offer on the introduction of the special educational license for using the special variant of COMSOL Multiphysics

software placed on a disk will be possible. In this case, it will allow reducing essentially the cost of a disk with the course and the software for the user.

As a result the user of the course will have an opportunity:

- During a study of the course, to use a computer modeling of the basic examples for better understanding and learning the separate positions of the course;

- To receive a good experience in a work with such modern computing means intended for modeling of the EMF phenomena and problems, as the software package COMSOL Multiphysics;

- To make the received knowledge and skills more firm by realization of examples and exercises with calculations and modeling.

2. Features of the course «Theory of electromagnetic field »

A basis of this electronic course is the standard course of lectures «Theory of the electromagnetic field», usually presented to the students studying the electrotechnical specialities in the Russian technical universities. The contents of the course includes five sections (General problems of the theory of the electromagnetic field (EMF), the Electrostatic field, the Electric field of a direct current in conducting medium, the Magnetostatic field and the Variable harmonic EMF). We have not include in structure of the course the section "The EMF in moving mediums" because this section usually does not a part of the standard student's course for these specialities

Each section shares on subsections devoted to the separate concrete themes of the course. The list of sections and subsections of the course is presented in the Appendix At start of the program-shell of a disk with the electronic course the **Table of contents of the course** menu containing all sections is displayed. When we select the proper section by a mouse, a menu with all subsections of this section will open. The menus can be preserved on the screen or to cause at any moment of work with the course. By a call of the subsection there is a first page of the caused subsection will appear and further the text of the course can be viewed page by page. On the display the information what page of what section and subsection is presented in the moment on the screen should be shown.

The page can be presented in two modes: with the normal size of a font or with the increased size for comfort reading.

The separate pages, the text of subsections or sections can be remembered in the form of files or to print. The user has no opportunity to edit the text of the course but

he can create some additional subsections in which the text is brought and edited by the user himself. The number of such user's subsections and their volume are limited. Such opportunities are very useful for lecturers in case when the concrete lecturer wishes to give the comments to the main text of the course or to include in it his own material specially prepared for his students.

By means of the mouse, it is possible to display the information with definition of the unfamiliar term or a unit of measure presented in the text on the current page. If in the text there is a reference to formulas or the figures located on other pages of the course, these references are made out as standard hyperlinks.

On the current screen of the course. it is also possible to show a brief information about the well-known scientists in honor of whom those or other laws, the phenomenon or physical units are named.

3. Exercises and the demonstrations included in the course.

The special Exercises and the demonstration examples explaining the separate positions of the course are included in the structure of the course as independent subsections These items are carried out by means of the COMSOL Multiphysics software.

The list of these special subsections is presented in a structure of the table of contents of the course (see Appendix). The user has an opportunity to model the different problems formulated in separate subsections of the course with application of various software of the COMSOL Multiphysics complex. In particular, it is possible to use programming in the COMSOL Script environment, modeling in PDE Modes, solving problems by means of electromagnetic applied modes of COMSOL Multiphysics nucleus and the Module of electromagnetism. Examples and exercises will allow students to acquire better the presented materials of the course and to receive very important skills of work with modern toolkit for modeling and solution of various EMF problems.

It is supposed, that the hypertext of the course and the version of the COMSOL Multiphysics program complex will arrange on one CDROM. Thus, the version of COMSOL Multiphysics will be specially adapted under the used demonstration examples and exercises. To model the next exercise or an example the necessary Comsol Multiphysics tool will be simply caused from the text of the course. The student will have an opportunity to change some conditions of a solved problem, to enter some new values of parameters and to use graphic and other means of a complex for output, postprocessing and visualization of the solution.

By using COMSOL Script, the user can write, debug and realize the programs in the COMSOL Script environment, and also memorizes them in the form of m- or mph-files for subsequent use. In the corresponding sections of the course, there are the specially prepared m-files which can be used also for various applications. Practically in the course, there are no functional restrictions on the use of COMSOL Script. Using of other means and modes of COMSOL Multiphysics are limited by the presented examples and demonstrations. In the process of solution of a corresponding problem, the user can memorize (or print) the statement of a problem, the name of a used mode, the parameters of this mode entered by the user, and the results of the solution (in the form of numerical values, figures and pictures). It is possible also to realize a small volume of comments. The texts of the presented examples are not edited by the user but he can change the values of the parameters allocated in the text and to carry out the modeling with the changed values.

4. Interaction of the COMSOL Multiphysics complex with the text of the course

The user finds the corresponding subsection concerning **Exercises** or **Demonstrations** in the menu of the course**.** After call of this subsection, the user will go to the necessary tool of COMSOL Multiphysics. On the screen he will see the Model Navigator window if the subsection is connected with the COMSOL Multiphysics models or the command window of COMSOL Script for the case of COMSOL Script using. Simultaneously, on the screen there will be a menu with the formulation of the problem and the strategy for its solution, and also the menu-help with the list of four modes which are represented on a choice to the user: the PDE modes (Coefficient Form and General Form), the mode of Comsol Multiphysics nucleus, and the mode of the Module of Electromagnetism. The user will choose a suitable mode and further, the menu-help will present him the instructions how to solve the given problem in that mode (steps which need to be made, values of coefficients and expressions which need to be substituted, and what result should turn out). Using this help, the user works with the standard menus of COMSOL Multiphysics and receives the necessary results on the screen. Thus, the user receives the solution for the given values of parameters. As it was mentioned earlier, he can change the values of some parameters and solve the problem with these

changed values. The user also has an opportunity to save the received results of solutions in the form of separate files with comments.

5. Two examples of Exercise and Demonstration subsections

Below we present two examples of exercises included in the given course.

Example 1. **Exercises. Calculation of the magnetic dipole moment vector of the spatial piecewise rectilinear contour with current in COMSOL Script. (Subsection 1.9}**

Problem

Consider a contour with electric current *I* represented in space as a perimeter of a triangle (see Fig.1). The Cartesian coordinates of the triangle vertexes are given: x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3. Here the lower indexes are the numbers of vertexes. The vertexes are numbered in a direction of the electric current flow (see arrows in the Fig. 1).

It is required to write the function of COMSOL Script which calculates the dipole magnetic moment of the contour. During the design of a m-file, it is possible to assume that the spatial coordinates of the contour are measured in meters and current in amperes. It is supposed to call the m-function prepared during the solution of the previous problem.
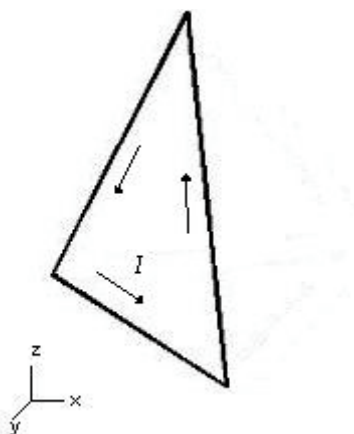

Fig. 1 A contour with electric current

Solution.

A vector of the triangle area in three-dimensional space is calculated by the formula:

$$S = \mathbf{1}_x \cdot \begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix} + \mathbf{1}_y \cdot \begin{vmatrix} 1 & z_1 & x_1 \\ 1 & z_2 & x_2 \\ 1 & z_3 & x_3 \end{vmatrix} + \mathbf{1}_z \cdot \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

where $\mathbf{1}_x$, $\mathbf{1}_y$, $\mathbf{1}_z$ are the basis vectors of the Cartesian system of coordinates.

The text of m-function is presented below.

% m_dip_moment – Calculation of magnetic dipole moment of the triangle contour with current in space
% pm = m_dip_moment(tok,nodes)

```
% INPUT PARAMETERS
% tok –  current in a contour;
% nodes – a square matrix given by [x1, x2, x3; y1, y2, y3; z1, z2, z3]' , in each line
of it, the coordinates of the corresponding vertex is written.
% OUTPUT PARAMETERS
% pm – a matrix row of the Cartesian components of the magnetic dipole moment.
function pm = m_dip_moment(tok,nodes);
pm=tok*[det([ones(3,1) nodes(:,[2,3])]) det([ones(3,1) nodes(:,[3,1])]) det([ones(3,1)
nodes(:,[1,2])])]/2;
% In the last operator a vector of the triangle area is multiplied by a current.
```

The example of a start of the developed m-function.

```
C>> nodes=10*rand(3)
nodes =
      9.5013      4.8598      4.5647
      2.3114      8.913      0.18504
      6.0684      7.621      8.2141
C>> pm=m_dip_moment(1,nodes)
pm =
      13.442      20.637      -2.9692
```

In this case we have received P_M = (13.442·1_x + 20.637·1_y - 2.9692·1_z) A·m^2, if current in the contour is equal to one ampere.

Example 2. Exercises. Analytical calculation and modeling of ESF and capacities of a coaxial cable with shifted core in COMSOL Multiphysics. (Subsection 2.18)

Problem

By means of the external source of the electromotive force (EMF) between a core and a shell of a coaxial cable the electric voltage is created. The geometry of cross-section of a cable is shown in Fig. 2.. The cable sheath radius is equal 5 mm. The cable core radius is equal 1 mm. The cable core center is shift with respect to the center of the cable sheath on 2.5 mm. The voltage between the sheath and the core is 10 V. The absolute permittivity of the dielectric is 8.85419E-3 pF/mm. It is required to calculate the distribution of the scalar electric potential in the dielectric and the cable capacity per unit of length.
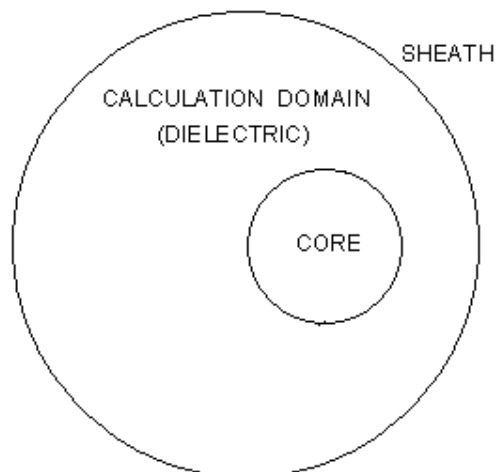

Fig. 2. Geometry of cross-section of a cable

Solution
For solution we use ... ric potential:

where e_a - the abs ... potential.
In the coaxial cabl ... the sheath) are
equipotential surfa ... nductors are the
boundaries of the c ... as follows:
... ary,
... ndary.

By modeling in PD ... following pictures
of the distribution o...
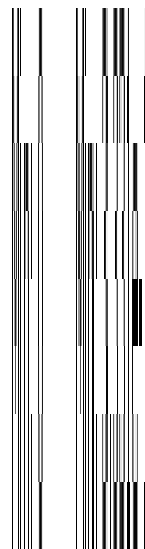
b) 3D picture with isolines

Fig. 3. The color graphs of the scalar electric potential distribution

By modeling in the electrostatic applied mode of COMSOL Multiphysics nucleus the user can receive the color graph of the absolute error of numerical calculation of the electric field
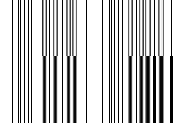
Fig.4 The absolute error of numerical calculation of the electric field

For calculation of the cable capacitance per unit length (pF/mm) we can use three approaches: analytical (ca), numerical on the field energy (ce) and numerical on the cable core charge (cq).

Below, we present a content of the COMSOL Script command window for the cable capacitance calculation:

```
C» format long
C» format compact
C» [ca0,ce0,cq0]=postinterp(fem,'ca0','ce0','cq0',[0;0])
ca0 =
   0.04271529728710
ce0 =
   0.04271537160628
cq0 =
   0.04268880924926
```

Appendix

Theory of the electromagnetic field.

Educational course on the basis of the program complex COMSOL Multiphysics

I. Common problems of the electromagnetic field theory

1. Definition of an electromagnetic field (EMF)
2. Brief performance of mathematical means of the EMF theory
3. Initial physical sources of EMF
4. Electric polarization and magnetization of matter
5. Volumetric charge density. Current density.
6. Definitions of vectors of the electric field intensity and the magnetic induction.
7. Definitions of vectors of the electric displacement and the magnetic field intensity in vacuum.
8. General definitions of vectors of the electric displacement and the magnetic field intensity
9. **Exercises.** Calculation of magnetic dipole moments of the spatial piecewise rectilinear contours with current in COMSOL Script.
10. EMF equations in the integral form:

11. EMF equations in the differential form (Maxwell's equations) for stationary mediums.
12. **Exercises.** Analysis of the given fields of the EMF vectors using the EMF equations in integral and differential forms on the base of COMSOL Script
13. Relations between the field vectors and physical properties of matter (the material connection equations).
14. **Exercises.** Construction of graphs of the material electrophysical performances on reference dates with help of COMSOL Script.
15. Energy of EMF. Power of thermal losses of energy during the passing of conductivity currents. Power of outside EMF sources.
16. **Exercises.** Energy analysis of the given EMF field vectors using COMSOL Script.
17. Boundary conditions for the EMF vectors on the boundary surfaces between mediums. Concept about the superficial rotor and the superficial divergence of the vector field.
18. Charge conservation law in the differential and the integral forms.
19. Boundary conditions for components of the current density on the medium boundary surfaces.
20. The Umov-Poynting theorem.
21. **Exercises.** Analysis of power fluxes in the given fields of the EMF vectors with use of COMSOL Script and the Umov-Poynting theorems.

II. Electrostatic field

1. Definition of an electrostatic field (ESF).
2. ESF equations in the integral and differential forms. Linearized equations of the material
   connection.
3. Boundary conditions for the ESF vectors on the boundary surfaces of dielectrics and on the surfaces of conductors.
4. Concept about a scalar electric potential.
5. The electrostatic equation with respect to the scalar electric potential.
6. Coefficient and general forms of the electrostatics equation in COMSOL Multiphysics.
7. Scalar boundary value problem of electrostatics. Types of boundary conditions (BC) on the boundary of the calculated domain. Representation of BC in COMSOL Multiphysics
8. Technology of construction of the ESF models in PDE Modes, in Electrostatic applied modes of COMSOL Multiphysics nucleus and in Electrostatic applied modes of Electromagnetics Module.
9. Energy of the system of charged conductors.
10. Technology of ESF energy calculation in COMSOL Multiphysics.
11. Fundamental solution of the scalar equation of electrostatics. Coulomb's Law as a consequence of this solution.
12. ESF of the electric dipole
13. **Exercises.** Calculation and visualization of ESF of the electric dipole in COMSOL Script.
14. ESF of the indefinitely long charged axis.
15. **Exercises.** Analytical calculations of ESF parameters for the indefinitely long coaxial system of conductors.
16. **Exercises.** Modeling ESF of coaxial systems in COMSOL Multiphysics in different one- and two-dimensional formulations.
17. ESF of two parallel charged axes with opposite signs of charges.

18. ESF and capacity of parallel cylinders with non coincident axes put one inside another.
19. **Demonstration.** Analytical calculation and modeling ESF and capacities of a coaxial cable with shifted core in different modes of COMSOL Multiphysics.

III. Electric field of a direct current in conducting medium

1. Laws of a constant electric field (CEF) in conducting medium in differential form.
2. Boundary conditions for the CEF vectors in conducting medium on the interfaces between media.
3. Analogy between CEF in conducting medium and ESF in dielectric.
4. Coefficient and general forms of the mathematical physics equation concerning the scalar electric potential. A boundary value problem of the CEF analysis.
5. Technology of construction of CEF in conducting medium models in PDE Modes, in   Electrostatic applied modes of COMSOL Multiphysics nucleus and in Electrostatic applied modes of Electromagnetism Module.
6. **Demonstration.** Calculations of electric fields of the current spreading of the ground connections (analytically and in COMSOL Multiphysics).

IV. Magnetostatic field

1. Lows of a magnetostatic field (MSF) in integral and differential forms.
2. Concept about magnetic poles.
3. Linearized forms of the material connection equations in magnetostatics
4. Energy of MSF.
5. Boundary conditions for the MSF vectors on the interfaces between media
6. Vector magnetic potential. A vector boundary value problem of magnetostatics.
7. Coefficient and general forms of the vector equation of magnetostatics in PDE Modes, in Magnetostatic applied modes of COMSOL Multiphysics nucleus and Electromagnetism Module. Technology of construction of the vector MSF models in COMSOL Multiphysics.
8. Integral parameters of MSF: magnetic fluxes, flux linkages, inductivities, cross inductivities.
9. **Exercises.** Analytical calculations of cross inductivities of circular and rectangular coaxial contours. Application of COMSOL Script.
10. **Exercises.** Solution of the vector boundary value problem of magnetostatics connected with the calculations of MSF for the cylindrical coil with and without magnetic core in COMSOL Multiphysics.
11. **Exercises.** Calculations of winding inductivities and cross inductivities by solution of the vector boundary value problem of magnetostatics in COMSOL Multiphysics.
12. **Demonstration.** Modeling of MSF in the cables protected and nonprotected by the magnetic shields in COMSOL Multiphysics.
13. Scalar magnetic potential. A scalar boundary value problem of magnetostatics.
14. Coefficient and general forms of the mathematical physics equation concerning the scalar magnetic potential. Construction of the applied modes in PDE Modes of COMSOL Multiphysics.

15. **Exercises.** MSF modeling of cylindrical coils in COMSOL Multiphysics based on the scalar boundary value problem of magnetostatics with calculations of integral parameters of the field.
16. Theory bases of the power interactions in magnetic fields.
17. **Demonstration.** Modeling of static angular characteristics of moments for switch machines with constant magnets and switch-inductor machines without constant magnets.
18. **Demonstration.** Modeling of force interaction of a curvilinear thin ferromagnetic console beam with MSF of the cylindrical coil.

V. Alternating harmonic EMF

1. Harmonic electromagnetic field (HEMF). The basic concepts and definitions. Complex representations of harmoniously changing vectors of EMF.
2. Maxwell's equations in the complex (spatial-frequency) form. The complex equations of material connection.
3. Umov-Pointing's theorem in the complex form
4. Complex parameters of medium electrophysical properties
5. Two classical systems of the EMF electrodynamic potentials.
6. System of mathematical physics equations concerning the vector magnetic potential and the scalar electric potential.
7. D'Alembert's vector and scalar spatially - frequency equations for a homogeneous medium with losses and without losses. The wave equations in the complex form. Connection of wave parameters with complex parameters of a medium.
8. **Demonstration.** Flat wave in a homogeneous dielectric. Animation of such wave in COMSOL Multiphysics.
9. Dual system of electrodynamic potentials. A dualism principle of differential statements of the analysis problems for HEMF.
10. A set of mathematical physics equations concerning the vector electric and the scalar magnetic potentials for an inhomogeneous medium.
11. Concept about superficial effect and effect of proximity.
12. **Demonstration.** Flat wave in a homogeneous conductor (conducting half-space). Taking into account of the displacement currents in a case of an incomplete dielectric. Animation of such wave in COMSOL Multiphysics.
13. Superficial effect in a conducting plate.
14. **Exercises.** Programming computing script of calculation of the superficial effect in a conducting plate in COMSOL Script. Realization of computing experiments with the investigation of the influence of the physical properties of the plate material on the superficial effect.
15. Superficial effect in a round wire.
16. **Exercises.** Programming computing script of calculation of the superficial effect in a round wire in COMSOL Script. Calculations of the frequency response characteristics of the wire complex resistance on unit of length at various values of the parameters of the physical properties of the wire material.
17. The review of physical applied modes of the harmonic EMF calculation in COMSOL Multiphysics (including also in *the* Module of Electromagnetism).

18. **Demonstration.** Modeling superficial effect in a round wire in COMSOL Multiphysics.
19. **Demonstration.** Modeling effect of proximity of two round wires in COMSOL Multiphysics.

20. **Demonstration.** Modeling the cylindrical EMFscreen in a cross harmonic magnetic field in COMSOL Multiphysics.